Practice Questions from HW 26

1. Consider the function $f(x) = \sum_{n=0}^{\infty} 30\left(\frac{x-40}{20}\right)^n = 30 + 30\left(\frac{x-40}{20}\right) + 30\left(\frac{x-40}{20}\right)^2 + 30\left(\frac{x-40}{20}\right)^3 + \ldots$

a. Evaluate. No work need be shown.
Write in the box an exact number or DNE or ∞ or $-\infty$.

$f(72) =$ ☐ $f(56) =$ ☐ $f(20) =$ ☐ $f(60) =$ ☐

b. For what values of x does $f(x)$ converge? Show work. ☐ $< x <$ ☐

c. Report the sum of the series on its interval of convergence.

d. What is true about the graph of $f(x)$ at the left endpoint? ______________________

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

e. What is true about the graph of $f(x)$ at the right endpoint? ______________________

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

2. The series $c(x) = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot 2x^{5k}}{1024^k}$ is a child of the geometric series $\sum_{k=0}^{\infty} ar^k$ where the value of $a =$ ☐ and ☐ $= r$

which converges for ☐ $< x <$ ☐. You can solve the inequality graphically or with a table.

a. At the left endpoint, $c(x)$ becomes the series $\sum_{k=0}^{\infty}$ ☐ = ☐ + ☐ + ☐ + ☐ + … which will ____________ .

{converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n =$ ☐. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $c(x)$ at the left endpoint? ______________________

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

b. At the right endpoint, $c(x)$ becomes the series $\sum_{k=0}^{\infty}$ ☐ = ☐ + ☐ + ☐ + ☐ + … which will ____________ .

{converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n =$ ☐. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $c(x)$ at the right endpoint? ______________________

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

c. Report the sum of the series on its interval of convergence.

3. Complete: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots =$ ☐. The name of this series is called the ______________________ series.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

4. Complete: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots =$ ☐. The name of this series is called the ______________________ series.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

5. **a.** In sigma notation the series $f(x) = 1 - x + (-x)^2 + (-x)^3 + (-x)^4 + \ldots = \sum^{\infty}_{\square} \square$

b. The series $f(x)$ is a child of the geometric series $\sum_{k=0}^{\infty} ar^k$ where the value of $a = \square$ and $\square = r$

The radius of convergence of $f(x)$ is $R =$ ____.

c. At the left endpoint, $f(x)$ becomes the series $\sum_{k=0}^{\infty} \square = \square + \square + \square + \square + \ldots$ which will ____________ .
{converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n = \square$. Write in the box an exact number or DNE or ∞ or −∞.

What is true about the graph of $f(x)$ at the left endpoint? ______________________________
{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

d. At the right endpoint, $f(x)$ becomes the series $\sum_{k=0}^{\infty} \square = \square + \square + \square + \square + \ldots$ which will ____________ .
{converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n = \square$. Write in the box an exact number or DNE or ∞ or −∞.

What is true about the graph of $f(x)$ at the right endpoint? ______________________________
{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

e. Report the sum of the series $f(x)$ on its interval of convergence.

f. Use the Root Test or Ratio Test (your choice) to show that the radius of convergence in part **b** is what you have claimed.

g. Integrate the above series term by term to create $g(x) = \int f(x)dx$. TIP: Be sure you are integrating.

$g(x) = \square + \square + \square + \square + \square + \ldots = \sum_{k=1}^{\infty} \square$

h. The radius of convergence of $g(x)$ is $R =$ ____.

i. At the left endpoint, $g(x)$ becomes the series $\sum_{k=1}^{\infty} \square = \square + \square + \square + \square + \ldots$ which will ____________ .
{converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n = \square$. Write in the box an exact number or DNE or ∞ or −∞.

What is true about the graph of $g(x)$ at the left endpoint? ______________________________
{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

j. At the right endpoint, $g(x)$ becomes the series $\sum_{k=1}^{\infty} \square = \square + \square + \square + \square + \ldots$ which will ____________ .
{converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n = \square$. Write in the box an exact number or DNE or ∞ or −∞.

What is true about the graph of $g(x)$ at the right endpoint? ______________________________
{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

k. Report the sum of the series $g(x)$ on its interval of convergence. TIP: Integrate the expression in **5e**.

6. **a.** In sigma notation the series $u(x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \dots = \sum_{\square}^{\infty} \square$

b. Use the series for $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ to find what function $u(x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \dots$ approximates.

$u(x) = \square$. The radius of convergence is $R = \square$

Simplified please.

c. Write the first four terms of the series for $u(x)$ in expanded form if $x = -1$. $\square - \dots$

The left endpoint $x = -1$ __________ in the interval of convergence. Explain your answer.
{is, is not}

Reason: ____

What is true about the graph of $u(x)$ at the left endpoint? ______________________________

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

d. Write the first four terms of the series for $u(x)$ in expanded form if $x = 1$. $\square - \dots$

The right endpoint $x = 1$ __________ in the interval of convergence. Explain your answer.
{is, is not}

Simplified please.

Reason: ____

What is true about the graph of $u(x)$ at the right endpoint? ______________________________

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

7. Consider the series $v(x) = 32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \dots$

a. Examine the pattern to report the next term: $32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \square$

b. The series $v(x)$ is a child series of the series $\tan^{-1} w = w - \frac{w^3}{3} + \frac{w^5}{5} - \frac{w^7}{7} + \dots = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{w^{2n+1}}{2n+1}$
which converges on $-1 \le w \le 1$. Use the series for $\tan^{-1} w$ *to* write what $v(x)$ converges to on its interval of convergence.

$v(x) = 32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \dots = \square$

c. Write the series $v(x)$ in sigma notation: $v(x) = \sum_{n=1}^{\infty} (-1)^n \cdot \square \cdot \frac{\left(\square\right)^{2n+1}}{2n+1}$

d. Differentiate the series $v(x)$ term by term to create the expanded series for $v'(x)$. TIP: Be sure you are differentiating.

$v'(x) = \square + \square + \square + \square + \dots$

e. The radius of convergence of $v'(x)$ is $R =$ ____. On its interval of convergence, $v'(x)$ converges to $v'(x) = \square$
TIP: Differentiate the expression in part **b.**

f. Write the series $v'(x)$ in sigma notation: $v'(x) = \sum_{n=1}^{\infty} \square$

TIP: Differentiate the expression in part **c.**

g. Write the first four terms of the series $v'(x)$ in expanded form if $x = -1$. [ ]

Simplified please.

The left endpoint $x = -1$ __________ {is, is not} in the interval of convergence. Explain your answer.

Reason: ____

What is true about the graph of $v'(x)$ at the left endpoint? ____

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

h. Write the first four terms of the series $v'(x)$ in expanded form if $x = 1$. [$-$] ...

Simplified please.

The right endpoint $x = 1$ __________ {is, is not} in the interval of convergence. Explain your answer.

Reason: ____

What is true about the graph of $v'(x)$ at the right endpoint? ____

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

d. Sketch a graph of the series $v'(x)$ on its the interval of convergence.

8. The term-by-term derivative of $f(x) = \sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ is the power series below.

a. Write the first four nonzero terms of the series for $f'(x)$.

Simplified please

$f'(x) =$ [ ] $+ \ldots$

b. The radius of convergence of $f'(x)$ is $R =$ ____

c. If x is equal to the **left endpoint** of the interval of convergence, the series for $f'(x)$ will ________________.
{converge, diverge}

e. If x is equal to the **right endpoint** of the interval of convergence, the series for $f'(x)$ will ________________.
{converge, diverge}

f. Write the series for $f'(x)$ in sigma notation.

$$f'(x) = \sum_{n=\square}^{\infty} \left(\square \right)$$

g. When x is in the interval of convergence, we can write the series for $f'(x)$ as what rational function?

$$f'(x) = \frac{\square}{\square}$$

h. Sketch a graph of $\sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ on its the interval of convergence.