Practice Questions from 11.1-11.4

1. Answer the following for the power series $\sum c_n(x-a)^n$. Complete the blanks.

a. The power series $\sum c_n(x-a)^n$ is centered at the value $x =$ ________.

b. Suppose the interval of convergence is **all real numbers**. Then the radius of convergence is $R =$ _______.

c. Suppose the interval of convergence is only **the value** $\boldsymbol{x = a}$. Then the radius of convergence is $R =$ _______.

d. Suppose the interval of convergence is $\boldsymbol{|x-a| < b}$, i.e. $\boldsymbol{a - b < x < a + b}$. Then the radius of convergence is $R =$ _______.

2. The interval of convergence of $\sum_{n=1}^{\infty}\left(\frac{x-4}{2}\right)^n$ is $\square < x < \square$. Show work below.

Hint: It is a geometric series.

3. Report the interval of convergence of $\sum_{n=0}^{\infty} n!x^{5n}$. Select one.

A. $-1 < x < 1$ B. $-\frac{1}{\sqrt{5}} < x < \frac{1}{\sqrt{5}}$ C. $-\sqrt[5]{5} < x < \sqrt[5]{5}$ D. $x = 0$ E. $-\frac{1}{5} < x < \frac{1}{5}$ F. $-\infty < x < \infty$

4. The interval of convergence of $\sum_{n=1}^{\infty}\frac{x^{3n}}{n!}$ is $\square < x < \square$. Show work below.

5. Consider $\sum_{n=1}^{\infty}\frac{(3x)^n}{n}$

a. The radius of convergence is $R =$ ______. Show work below.

b. If x is equal to the **left endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty}\frac{(3x)^n}{n}$ will _________________.
{converge, diverge}

c. If x is equal to the **right endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty}\frac{(3x)^n}{n}$ will_________________.
{converge, diverge}

d. State the **reasons** which justify your claims about the endpoints in parts **b** and **c**.

6. Consider $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$

a. The radius of convergence is $R =$ ______. Show work below.

b. If x is equal to the **left endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$ will ________________.
{converge, diverge}

c. If x is equal to the **right endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$ will________________.
{converge, diverge}

d. State the **reasons** which justify your claims about the endpoints in parts **b** and **c**.

Fun Facts:

For all x we have $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots$ $\qquad \sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$ $\qquad \cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$

For $-1 < x < 1$ we have $\frac{1}{1-x} = \sum_{n=0}^{\infty}x^n = 1+x+x^2+x^3+x^4+\cdots$

For $-1 < x \le 1$ we have $\ln(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$ $\qquad$ For $-1 \le x \le 1$ we have $\tan^{-1}x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$

7. Complete: $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots =$ ☐. The name of this series is called the ____________________________ series.

TIP: Use a Fun Fact above.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

8. Complete: $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\cdots =$ ☐. The name of this series is called the ____________________________ series.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

9. **a.** In sigma notation the series $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots = \sum_{\square}^{\infty} \square$

b. Use one of the Fun Facts above to determine what function $f(x)$ the series $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots$ approximates.

$f(x) = \square$. .The radius of convergence is $R = \square$

Simplified please.

c. Write the first four terms of the series in expanded form if $x = -1$. $\square$ $- \ldots$

The left endpoint $x = -1$ ____________ in the interval of convergence. Explain your answer.
{is, is not}

Reason: ____

Simplified please.

d. Write the first four terms of the series in expanded form if $x = 1$. $\square$ $- \ldots$

The right endpoint $x = 1$ ____________ in the interval of convergence. Explain your answer.
{is, is not}

Reason: ____

e. The term-by-term derivative of $f(x) = \sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ is the power series below.

a. Write the first four nonzero terms of the series for $f'(x)$.

Simplified please

$f'(x) = \square + \ldots$

b. The radius of convergence of $f'(x)$ is $R =$ ____

c. If x is equal to the **left endpoint** of the interval of convergence, the series for $f'(x)$ will ________________.
{converge, diverge}

d. If x is equal to the **right endpoint** of the interval of convergence, the series for $f'(x)$ will________________.
{converge, diverge}

e. Write the series for $f'(x)$ in sigma notation.

$f'(x) = \sum_{n=\square}^{\infty} \left(\square \right)$

f. When x is in the interval of convergence, we can write the series for $f'(x)$ as what rational function?

$f'(x) = \frac{\square}{\square}$

Additional Questions coming for 11.3 and 11.4. Meanwhile, see HW 25