## Practice Questions from 11.1-11.4

1. Answer the following for the power series  $\sum c_n (x-a)^n$ . Complete the blanks.

- **a.** The power series  $\sum c_n (x-a)^n$  is centered at the value x = \_\_\_\_\_
- **b.** Suppose the interval of convergence is **all real numbers**. Then the radius of convergence is R =\_\_\_\_\_.
- **c.** Suppose the interval of convergence is only the value x = a. Then the radius of convergence is  $R = \_$ .
- **d.** Suppose the interval of convergence is |x a| < b, i.e. a b < x < a + b. Then the radius of convergence is R =\_\_\_\_\_.
- 2. The interval of convergence of  $\sum_{n=1}^{\infty} \left(\frac{x-4}{2}\right)^n$  is  $\left| \frac{x}{2} \right|^n < x < \left| \frac{x}{2} \right|^n$ . Show work below. Hint: It is a geometric series.

3. Report the interval of convergence of  $\sum_{n=0}^{\infty} n! x^{5n}$ . Select one. A. -1 < x < 1 B.  $-\frac{1}{\sqrt{5}} < x < \frac{1}{\sqrt{5}}$  C.  $-\sqrt[5]{5} < x < \sqrt[5]{5}$  D. x = 0 E.  $-\frac{1}{5} < x < \frac{1}{5}$  F.  $-\infty < x < \infty$ 

4. The interval of convergence of 
$$\sum_{n=1}^{\infty} \frac{x^{3n}}{n!}$$
 is  $| < x < |$ . Show work below.

- 5. Consider  $\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$ 
  - **a**. The radius of convergence is R =\_\_\_\_\_. Show work below.
  - **b.** If x is equal to the **left endpoint** of the interval of convergence, the series  $\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$  will  $\frac{1}{\{\text{converge, diverge}\}}$ .

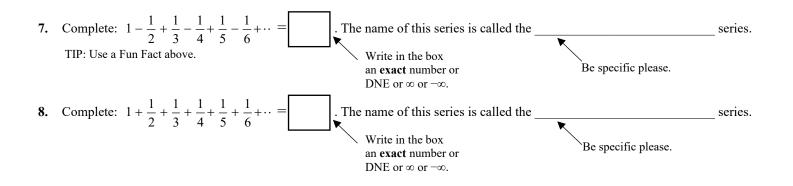
  - d. State the reasons which justify your claims about the endpoints in parts b and c.

- 6. Consider  $\sum_{n=1}^{\infty} (-1)^{n+1} \left( \frac{x^{n+11}}{n^2} \right)$ 
  - **a**. The radius of convergence is R =\_\_\_\_\_. Show work below.

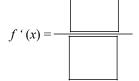
- **b.** If x is equal to the **left endpoint** of the interval of convergence, the series  $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{x^{n+1}}{n^2}\right)$  will  $\frac{1}{\{\text{converge, diverge}\}}$
- c. If x is equal to the **right endpoint** of the interval of convergence, the series  $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{x^{n+11}}{n^2}\right)$  will  $\underline{\qquad}$
- d. State the reasons which justify your claims about the endpoints in parts b and c.

Fun Facts:

For all x we have 
$$e^{x} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \frac{x^{4}}{4!} + \cdots$$
  $\sin x = x - \frac{x^{3}}{3!} + \frac{x^{5}}{5!} - \frac{x^{7}}{7!} + \cdots$   $\cos x = 1 - \frac{x^{2}}{2!} + \frac{x^{4}}{4!} - \frac{x^{6}}{6!} + \cdots$   
For  $-1 < x < 1$  we have  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^{n} = 1 + x + x^{2} + x^{3} + x^{4} + \cdots$   
For  $-1 < x \le 1$  we have  $\ln(1+x) = x - \frac{x^{2}}{2} + \frac{x^{3}}{3} - \frac{x^{4}}{4} + \cdots$  For  $-1 \le x \le 1$  we have  $\tan^{-1}x = x - \frac{x^{3}}{3} + \frac{x^{5}}{5} - \frac{x^{7}}{7} + \cdots$ 



9.	a.	In sigma notation the series $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \dots = \sum_{n=1}^{\infty}$
	b.	Use one of the Fun Facts above to determine what function $f(x)$ the series $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots$ approximates.
		f(x) = The radius of convergence is $R =$ Simplified please.
	c.	Write the first four terms of the series in expanded form if $x = -1$ .
		The left endpoint $x = -1$ in the interval of convergence. Explain your answer.
		Reason: Simplified please.
	d.	Write the first four terms of the series in expanded form if $x = 1$ .
		The right endpoint $x = 1$ in the interval of convergence. Explain your answer.
		Reason:
	e.	The term-by-term derivative of $f(x) = \sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ is the power series below.
	a.	Write the first four nonzero terms of the series for $f'(x)$ . Simplified please
		$f'(x) = + \dots$
	b.	The radius of convergence of $f'(x)$ is $R = $
	c.	If x is equal to the <b>left endpoint</b> of the interval of convergence, the series for $f'(x)$ will
	d.	If x is equal to the <b>right endpoint</b> of the interval of convergence, the series for $f'(x)$ will
	e.	Write the series for $f'(x)$ in sigma notation.
		$f^{\prime}(x) = \sum_{n=1}^{\infty} \left( \boxed{} \right)$
	f.	When x is in the interval of convergence, we can write the series for $f'(x)$ as what rational function?



Additional Questions coming for 11.3 and 11.4. Meanwhile, see HW 25