Practice Questions from 10.6-10.8

1. Suppose $\sum_{n=1}^{\infty}(-1)^{n+1}a_n = \frac{2}{1} - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \frac{6}{5} - \frac{7}{6} + \ldots$

a. What is a_n? $a_n = \boxed{}$

b. The series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ will ____.
{converge absolutely, converge conditionally, diverge }

c. Give a reason for your claim in part **b**.

2. Each alternating series below converges by the Alternating Series Test (AST).
Determine if the convergence is conditional or absolute.

a. $\sum_{n=1}^{\infty}\frac{(-1)^n 7n}{4n^3-3}$ will converge ____________________ because
{ absolutely, conditionally }

the series $\sum_{n=1}^{\infty}\frac{7n}{4n^3-3}$ will ________________ by the _______________________________ with b $\boxed{}$.
{converge, diverge} {Comparison Test, Limit Comparison Test}

Provide the details of your claim below.

b. $\sum_{n=1}^{\infty}(-1)^{n+1}a_n = \frac{1}{15} - \frac{1}{20} + \frac{1}{25} - \frac{1}{30} + \frac{1}{35} - \frac{1}{40} + \ldots$ will converge ____________________ because
{ absolutely, conditionally }

the series $\sum_{n=1}^{\infty}a_n = \sum_{n=1}^{\infty}\boxed{}$ will ________________ by the _______________________________ with $b_n = \boxed{}$.
{converge, diverge} {Comparison Test, Limit Comparison Test}

Provide the details of your claim below.

3. Report the two conditions for an alternating series $\sum_{n=1}^{\infty}b_n = \sum_{n=1}^{\infty}(-1)^{n+1}a_n$ to converge, where a_n is positive for all n.

i. ____

ii. ____

4. Give an example of any kind of divergent alternating series. Hint: Think about your answer to Question 3.

5. Give an example of an alternating series with the property that its nth term approaches 0 but it still diverges. There are many correct answers. Hint: Think about your answer to Question 3. You may write it in long form (expanded form) or use sigma notation, but use correct notation.

6. The Ratio Test and Root Test are based on the properties of convergence of
A. a p-series, $p\neq 1$. B. the harmonic series C. the alternating series D. a television series E. the world series. F. a geometric series

7. Which of these will help you determine if the series $\sum_{n=0}^{\infty} 2e^n$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
D. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
E. integral test F. ratio test G. nth Term Test for Divergence

8. Which of these will help you determine if the series $\sum_{n=0}^{\infty} e^{-2n}$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
D. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
E. integral test F. ratio test G. nth Term Test for Divergence

9. Which of these will help you determine if the series $\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1}}{n^2}\right)$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
D. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
E. ratio test F. nth Term Test for Divergence

10. Which of these will help you determine if the series $\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1}}{\sqrt{n}}\right)$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
D. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
E. ratio test F. nth Term Test for Divergence

11. Which of these will help you determine if the series $\sum_{n=1}^{\infty}\left(\frac{n+2}{n!}\right)$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
D. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
E. integral test F. ratio test G. nth Term Test for Divergence

12. Use the Ratio Test for each.

a. The series $\sum_{n=1}^{\infty}\frac{(-2)^n}{n!}$ will ______________ {converge, diverge} by the Ratio Test because $\lim_{n\to\infty} \boxed{} = \boxed{}$

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

b. The series $\sum_{n=1}^{\infty}\frac{4^n}{n^{800}}$ will ______________ {converge, diverge} by the Ratio Test because $\lim_{n\to\infty} \boxed{} = \boxed{}$.

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

8. Use the Root Test for each.

a. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\dfrac{8n^4}{7n^4+n+5}\right)^n$ will ______________ by the Root Test because $\lim_{n\to\infty}\boxed{} = \boxed{}$
{converge, diverge}

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

b. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\dfrac{2n}{3n+2}\right)^n$ will ______________ by the Root Test because $\lim_{n\to\infty}\boxed{} = \boxed{}$.
{converge, diverge}

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

c. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\dfrac{n+1}{n}\right)^{n^2}$ will ______________ by the Root Test because $\lim_{n\to\infty}\boxed{} = \boxed{}$.
{converge, diverge}

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

9. Consider the series $\sum_{n=1}^{\infty}\left(1+\dfrac{a}{n}\right)^{18n}$ for some real number a.

a. The series will ________________ .
{converge, diverge}

b. Circle the best answer to determine part **a**.
A. It is a p-series. B. It is a geometric series C. Use the Ratio Test D. Use the Root Test
E. Use the nth Term Test for Divergence

c. Explain more fully below how part **b** justifies part **a**.

10. Consider the series $\sum_{n=1}^{\infty}(-2)^n$

a. The series will ________________ .
{converge, diverge}

b. Which of these will help you determine if the series $\sum_{n=1}^{\infty}(-2)^n$ converges or diverges? Select all possible answers.

A. It is a p-series. B. It is a geometric series C. Use the Ratio Test D. Use the Root Test
E. Use the nth Term Test for Divergence

c. Explain more fully below how part **b** justifies part **a** for each of your choices.

11. Consider the series $\sum_{n=1}^{\infty}n(-0.5)^n$

a. The series will ________________ .
{converge, diverge}

b. Justify your claim in part a.