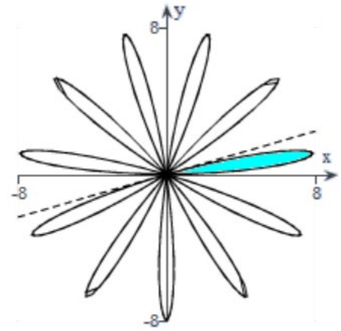
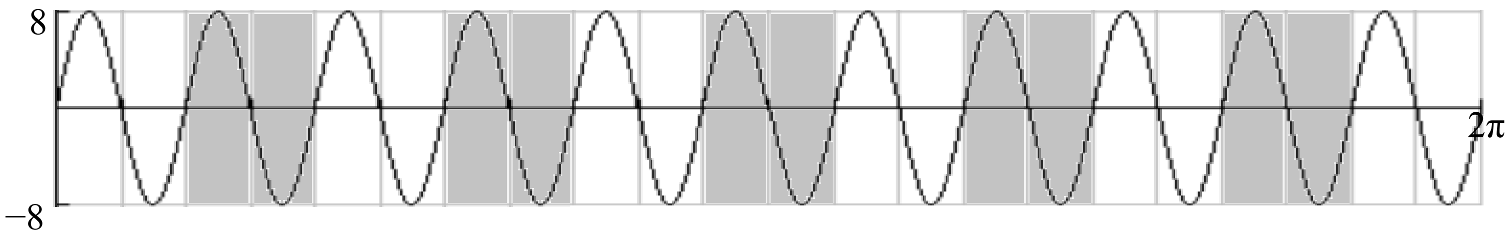
Area of a Polar Rose



Use the area formula $\int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta$ to find the area of one petal of the rose $r = 8\sin 11\theta$ shown to the right. Hint: The first petal starts and ends when $r = f(\theta) = 8\sin 11\theta = 0$.

The graph of $r = 8\sin 11\theta$ has ___ complete cycles in the interval $0 \le \theta \le 2\pi$.



1. To find the integration limits, we can find values $\theta = \alpha$ and $\theta = \beta$ for which $r = 8\sin 11\theta = 0$, since this will be where the first petal starts and ends. The graph starts at $\alpha = 0$. Find β for which the graph of $r = 9 \sin 11\theta = 0$.

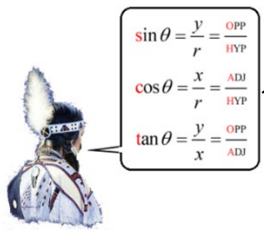
   $\beta =$ _________. (Report β as an exact value involving π.)

   TIP: The dashed line in the above graph is the polar equation $\theta = \beta$. You can enter in your polar grapher $r = 9 \sin 11\theta$ and θmin = 0 and θmax = β to check that you have sketched only the first petal.

2. The exact area is $\int_{0}^{\beta} \frac{1}{2} r^2 d\theta$ = __________ (Write as a multiple of π. Use FNINT)

➤ Recall from Section 12.2 that to convert from polar to Cartesian coordinates we use the relations

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta.$$



Arc Length of a Polar Curve

We now answer the arc length question for polar curves: Given the polar equation $r = f(\theta)$, what is the length of the corresponding curve for $\alpha \le \theta \le \beta$ (assuming the curve does not retrace itself on this interval)? The key idea is to express the polar equation as a set of parametric equations in Cartesian coordinates and then use the arc length formula for parametric equations derived in Section 12.1. Letting θ play the role of a parameter and using $r = f(\theta)$, parametric equations for the polar curve are

$$x = r\cos\theta = f(\theta)\cos\theta \quad \text{and} \quad y = r\sin\theta = f(\theta)\sin\theta,$$

where $\alpha \le \theta \le \beta$. The arc length formula in terms of the parameter θ is

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta,$$

where

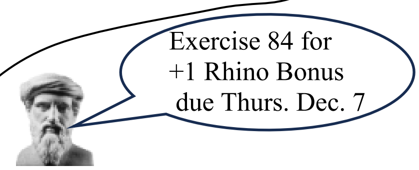
$$\frac{dx}{d\theta} = f'(\theta)\cos\theta - f(\theta)\sin\theta \quad \text{and} \quad \frac{dy}{d\theta} = f'(\theta)\sin\theta + f(\theta)\cos\theta.$$

When substituted into the arc length formula and simplified, the result is a new arc length integral (Exercise 84).

Arc Length of a Polar Curve

Let f have a continuous derivative on the interval $[\alpha, \beta]$. The **arc length** of the polar curve $r = f(\theta)$ on $[\alpha, \beta]$ is

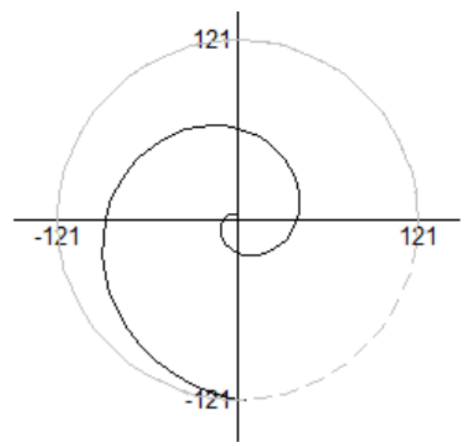
$$L = \int_\alpha^\beta \sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta.$$



1. The arc length from $\theta = 0$ to $\theta = 10.9$ of the polar spiral $r = \theta^2$ shown is given by $\int_0^{10.9} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Calculate the arc length correct to the nearest whole number.
You can use the FNINT command. Round to the nearest whole number.

$$\int_0^{10.9} \sqrt{\boxed{}}\, d\theta \approx \boxed{}$$



2. The arc length from $\theta = 0$ to $\theta = 2\pi$ of the cardioid $r = 4 - 4\sin\theta$ shown is given by $\int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Calculate the arc length. You can use the FNINT command.

$$\int_0^{2\pi} \sqrt{\boxed{}}\, d\theta = \boxed{}$$