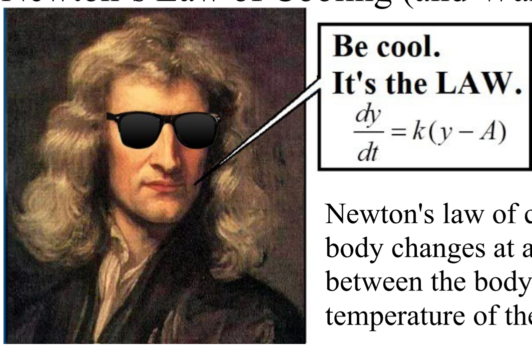
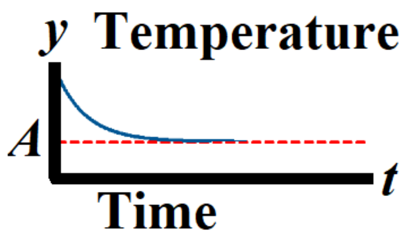
Newton's Law of Cooling (and Warming)

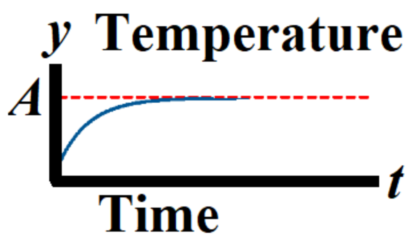


Newton's law of cooling (or warming) states that the temperature of a body changes at a rate proportional to the difference in temperature between the body and its surroundings, assuming the ambient temperature of the environment remains constant.

Another way to say this: *The closer a cooling body (such as hot coffee) gets to room temperature, the slower it cools.*



For warming: *The closer a warming body (such as ice water) gets to room temperature, the slower it warms.*



1. Using the graphs, explain why Cool Newt's differential equation $\frac{dy}{dt} = k(y - A)$ means the same thing as what is written in italics above.

2. Use the above graphs to fill in the blanks, assuming we have $\frac{dy}{dt} = k(y - A)$.

a. For a hot coffee cooling to a room temperature of A degrees, $\frac{dy}{dt}$ is ______________ {positive, negative}, the difference $y - A$ is ______________ {positive, negative}, and so the constant of proportionality k is ______________ {positive, negative}

b. For an ice water warming to a room temperature of A degrees, $\frac{dy}{dt}$ is ______________ {positive, negative}, the difference $y - A$ is ______________ {positive, negative}, and so the constant of proportionality k is ______________ {positive, negative}

3. Cool Newt can also write it as $\frac{dy}{dt} = k(A - y)$.

a. For a hot coffee, $\frac{dy}{dt}$ is ______________ {positive, negative}, $A - y$ is ______________ {positive, negative}, and k is ______________ {positive, negative}

b. For an ice water, $\frac{dy}{dt}$ is ______________ {positive, negative}, $A - y$ is ______________ {positive, negative}, and k is ______________ {positive, negative}

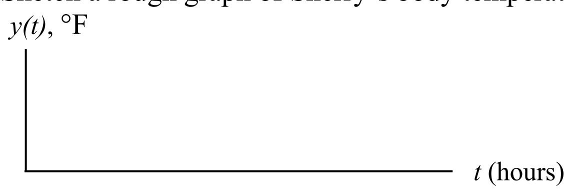
Count Dracula is thirsty for a fine Sherry, so he heads to the wine cellar. Turn the page over to see more details.

4. Alas, Sherry the wine steward meets her untimely end. (Dracula likes his Sherry chilled.)
Assume Sherry keeps the wine cellar at a constant temperature of 60°F.

a. If we use the equation, $\frac{dy}{dt} = k(y - A)$, then $A =$ _____. We expect k to be ____________.
{ positive, negative}

b. When Sherry is later discovered, sprawled face down on the wine cellar floor, she has cooled to a tepid 82˚F. We will assign this time of discovery to be $t = 0$ hours.

i. Sketch a rough graph of Sherry's body temperature. Label the y-axis with numbers.



ii. After several measurements of Sherry's body temperature, it was determined that when her body was 80˚F, her temperature was decreasing at a rate of 7 ˚F per hour. Find k. Show your calculations.

$k = \boxed{}$

iii. Complete the boxes to report how Sherry is chilling: $\frac{dy}{dt} = \boxed{}\left(\boxed{} - \boxed{}\right)$

c. Suppose that at the time of death, Sherry's temperature was 98.6°F. Complete the blank.
At the time of death, Sherry's body temperature was decreasing at ___________°F per hour.

d. Solve the differential equation in part **b.iii**. Complete the boxes with exact values.

$$y = \boxed{}\, e^{\boxed{}\, t} + \boxed{}$$

e. Complete:

i. $\lim_{t \to \infty} y(t) =$ ______.

ii. Report what this number represents in the context of this problem.

5. Differentiate your answer in part **4d** to show that $\frac{dy}{dt}$ is equivalent to what you reported in **4b.iii.**

6. How many minutes would it take for Sherry's body to reach 96°F after she was dispatched?
______ minutes (Report a whole number.) Explain your reasoning. Show your calculations.

(+0.5) Sink your teeth into this Rhino Participation Bonus:
Because Sherry works in a remote Transylvanian castle, it takes a very long time for the authorities to arrive.

The Count commands his servant, Renfield, to keep Sherry chilling in the wine cellar
until her rate of cooling is decreasing at a mere 0.14°F per hour.

a. Assume Sherry was discovered at precisely 10:00 A.M.
What clock time should Renfield summon his master for a chilled Sherry to meet the above requirements?

☐☐:☐☐ __.M.

Enter an **A** *or a* **P** *in the blank.*

You will need to complete part **b** correctly to earn any credit for part **a**.

b. Dracula requires Renfield to justify that part **a** is the precise clock time
by reporting the ***exact*** number of hours that Sherry must be chilling after her time of discovery.
You will need to solve an equation ***algebraically***. Show your work.

$t =$ ☐ hrs