## Lateral Surface Area

The lateral (side) surface area of a solid of revolution includes only its side, not its top or bottom. For example, the lateral surface area of a soup can is the area of the label.


1. Complete the boxes to report the lateral surface area of a soup can with radius $r$ and length $L$ from top to bottom.
Lateral Surface Area $=\square$

2. Report the arc length $L$ of any function $y=f(x)$ from $x=a$ to $x=b$.

3. Report the lateral surface area LSA created by revolving any continuous, differentiable function $y=f(x)$ about the $x$-axis from $x=a$ to $x=b$, assuming $f^{\prime}(x)$ is continuous.

4. Heinz is shown below with his water dish.

Use the FTC to set up and find its exact lateral surface area. Use the FTC to find the exact volume.



Cool fact: The lateral surface area LSA created by revolving any continuous, differentiable function $x=g(y)$ about the $y$-axis from $x=c$ to $x=d$, assuming $g^{\prime}(y)$ is continuous would be

$$
\int_{c}^{d} 2 \pi g(y) \sqrt{1+\left(g^{\prime}(y)\right)^{2}} d y=\int_{c}^{d} 2 \pi g(y) \sqrt{1+\left(\frac{d x}{d y}\right)^{2}} d y
$$

Rhino Participation Bonus (+2): Set up a definite integral and use the FTC to show the LSA of the cone is LSA $=\pi r L$ and the volume is $V=\frac{1}{3} \pi r^{2} h$. You can integrate with respect to $x$ or $y$.


