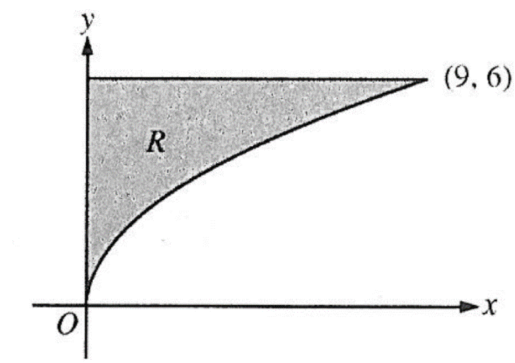
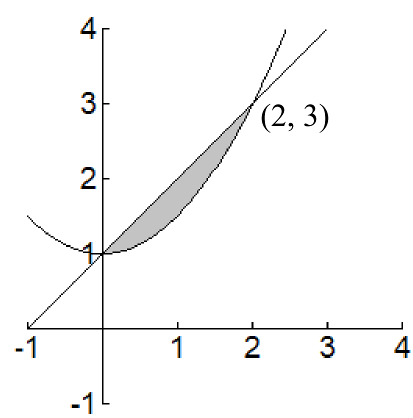
What Happens When the Smog Lifts in Los Angeles, California?

Calculate the volume and find the answer at the bottom of the page. Cross out the letter above each correct answer. When you finish, the answer to the title question will remain.

1. Region R is bounded by the y-axis, the line $y = 6$, and the curve $y = 2\sqrt{x}$.
Find the volume of the solid generated when Region R is rotated around
  a. The y-axis
  b. The x-axis
  c. The vertical line $x = -2$
  d. The horizontal line $y = 6$

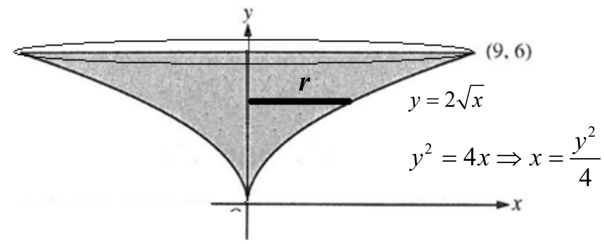


2. Let J be the region bounded by $f(x) = x + 1$ and $g(x) = \frac{1}{2}x^2 + 1$.
Find the volume of the solid generated when J is rotated around
  a. The line $y = 1$
  b. The line $x = 3$
  c. The x-axis
  d. The y-axis



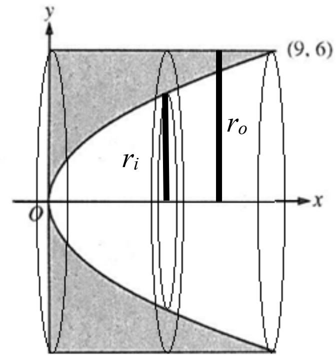
A	B	U	S	O	C	E	L	W	R	A	Y
54π	$\frac{8\pi}{3}$	$\frac{11\pi}{6}$	162π	2.4π	20.2π	97.2π	$\frac{32\pi}{3}$	$\frac{4\pi}{3}$	$\frac{16\pi}{15}$	1.2π	169.2π

KEY



1a. About the y-axis. Use the disk method. Here $r = x = \frac{y^2}{4}$.

$$V = \int_0^6 \pi(r)^2\,dy = \int_0^6 \pi(\tfrac{y^2}{4})^2\,dy = \int_0^6 \pi\tfrac{y^4}{16}\,dy = 97.2\pi$$



1b. About the x-axis. Use the washer method.

Here $r_o = 6$ and $r_i = y = 2\sqrt{x}$

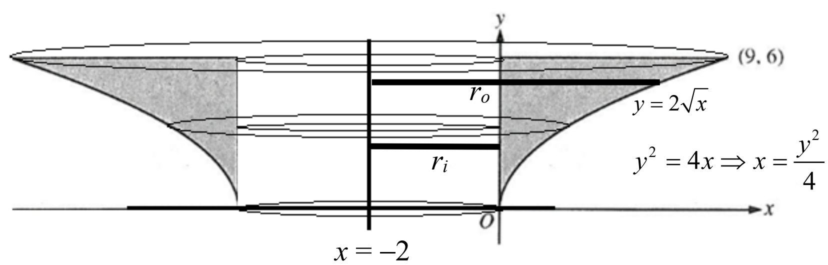
$$V = \int_0^9 \pi(r_o)^2\,dx - \int_0^9 \pi(r_i)^2\,dx$$
$$= \int_0^9 \pi(6)^2\,dx - \int_0^9 \pi(2\sqrt{x})^2\,dx = \pi\int_0^9 (36-4x)\,dx =$$
$$= 162\pi$$

1c. About the vertical line $x = -2$.

Use the washer method.

Here $r_o = 2 + \frac{y^2}{4}$ and $r_i = 2$

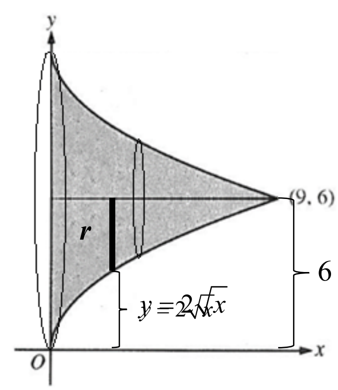
$$V = \int_0^6 \pi(r_o)^2\,dy - \int_0^6 \pi(r_i)^2\,dy$$
$$= \int_0^6 \pi(2+\tfrac{y^2}{4})^2\,dy - \int_0^6 \pi(2)^2\,dy = 169.2\pi$$



1d. The horizontal line $y = 6$.

Use the disk method. Here $r + 2\sqrt{x} = 6$ so $r = 6 - 2\sqrt{x}$.

$$V = \int_0^9 \pi(r)^2\,dx = \int_0^9 \pi(6-2\sqrt{x})^2\,dx = 54\pi$$



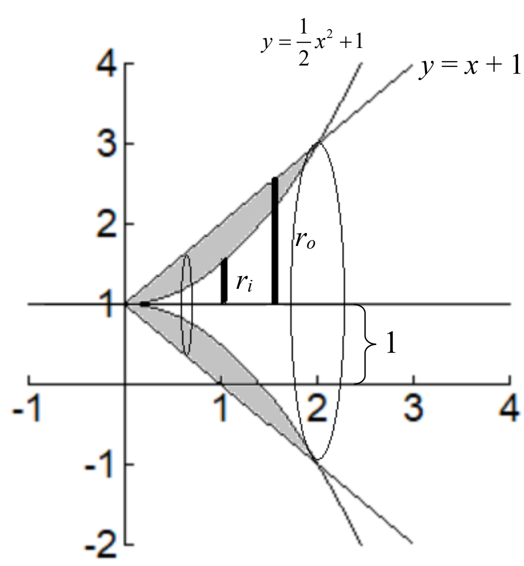
2a. About the line $y = 1$.

Use the washer method.

Here $r_o + 1 = x + 1$ so $r_o = x + 1 - 1 = x$.

$r_i + 1 = \frac{1}{2}x^2 + 1$ so $r_i = \frac{1}{2}x^2 + 1 - 1 = \frac{1}{2}x^2$.

$$V = \int_0^2 \pi(r_o)^2\,dx - \int_0^2 \pi(r_i)^2\,dx$$
$$= \int_0^2 \pi(x)^2\,dx - \int_0^2 \pi(\tfrac{1}{2}x^2)^2\,dx = \int_0^2 \pi(x^2 - \tfrac{1}{4}x^4)\,dx = \frac{16\pi}{15}$$



2b. About the line $x = 3$.

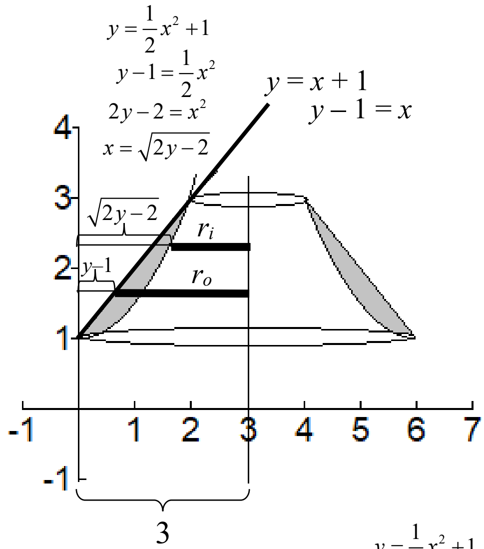
Use the washer method.

Here $r_o + y - 1 = 3$ so $r_o = 3 - (y - 1) = 4 - y$.

$r_i + \sqrt{2y-2} = 3$ so $r_i = 3 - \sqrt{2y-2}$.

$$V = \int_1^3 \pi (r_o)^2\,dy - \int_1^3 \pi (r_i)^2\,dy$$

$$= \int_1^3 \pi (4-y)^2\,dy - \int_1^3 \pi (3-\sqrt{2y-2})^2\,dy = \frac{26\pi}{3} - 6 = \frac{8\pi}{3}.$$



2c. About the x-axis

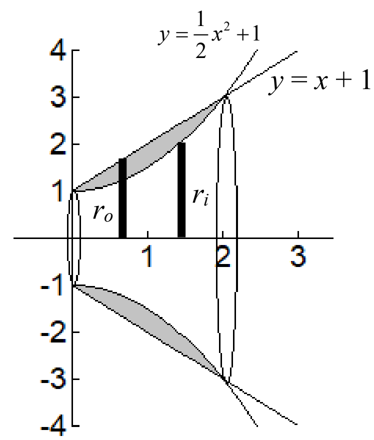
Use the washer method.

Here $r_o = x + 1$.

$r_i = \frac{1}{2}x^2 + 1$.

$$V = \int_0^2 \pi (r_o)^2\,dx - \int_0^2 \pi (r_i)^2\,dx$$

$$= \int_0^2 \pi (x+1)^2\,dx - \int_0^2 \pi (\tfrac{1}{2}x^2 + 1)^2\,dx = 2.4\pi.$$



2d. About the y-axis

Use the washer method.

Here $r_o = \sqrt{2y-2}$ (See 2b)

$r_i = y - 1$

$$V = \int_1^3 \pi (r_o)^2\,dy - \int_1^3 \pi (r_i)^2\,dy$$

$$= \int_1^3 \pi (\sqrt{2y-2})^2\,dy - \int_1^3 \pi (y-1)^2\,dy = \pi \int_1^3 (2y - 2 - (y-1)^2)\,dy$$

$$= \frac{4\pi}{3}$$