KEY MA 16600 Practice Questions over 12.2-12.3

1\. a First use the polar grid to find r and θ, *which are not unique.*

A. $r =$ 5, $\theta =$ π/6 and $x =$ _____, $y =$ _____

B. $r =$ 4, $\theta =$ π/3 and $x =$ _____, $y =$ _____

C. $r =$ 3, $\theta =$ π/2 and $x =$ _____, $y =$ _____

D. $r =$ 2, $\theta =$ 5π/6 and $x =$ _____, $y =$ _____

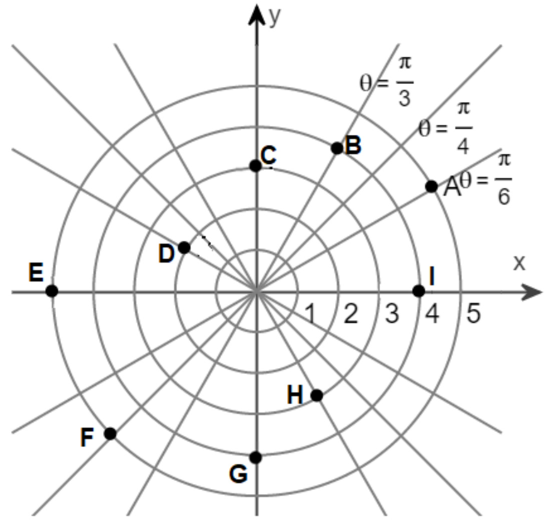
E. $r =$ 5, $\theta =$ π and $x =$ _____, $y =$ _____

F. $r =$ 5, $\theta =$ 5π/4 and $x =$ _____, $y =$ _____

G. $r =$ 4, $\theta =$ 3π/2 and $x =$ _____, $y =$ _____

H. $r =$ 3, $\theta =$ 5π/3 and $x =$ _____, $y =$ _____

I. $r =$ 4, $\theta =$ 0 and $x =$ _____, $y =$ _____



For θ please choose from
0, π/6, π/4, π/3, π/2, 2π/3, 3π/4, 5π/6, π,
7π/6, 5π/4, 4π/3, 3π/2, 5π/3, 7π/4, or 11π/6

Then stretch the unit circle to find exact values of x and y.

A. $r =$ 5, $\theta =$ π/6 and $x = \frac{5\sqrt{3}}{2}$, $y = \frac{5}{2}$

B. $r =$ 4, $\theta =$ π/3 and $x =$ 2, $y = 2\sqrt{3}$

C. $r =$ 3, $\theta =$ π/2 and $x =$ 0, $y =$ 3

D. $r =$ 2, $\theta =$ 5π/6 and $x = -\sqrt{3}$, $y =$ 1

E. $r =$ 5, $\theta =$ π and $x =$ −5, $y =$ 0

F. $r =$ 5, $\theta =$ 5π/4 and $x = -\frac{5\sqrt{2}}{2}$, $y = -\frac{5\sqrt{2}}{2}$

G. $r =$ 4, $\theta =$ 3π/2 and $x =$ 0, $y =$ −4

H. $r =$ 3, $\theta =$ 5π/3 and $x = \frac{3}{2}$, $y = -\frac{3\sqrt{3}}{2}$

I. $r =$ 4, $\theta =$ 0 and $x =$ 4, $y =$ 0

2. a. $2r = \csc\theta$

$r = \frac{1}{2} \cdot \frac{1}{\sin\theta}$ Divide both sides by 2

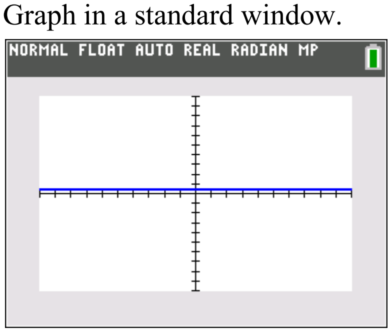
$r\sin\theta = \frac{1}{2}$ Multiply both sides by $\sin\theta$
Goal: Create $r\sin\theta$ to replace it with y

$$\boxed{y = \frac{1}{2}}$$

Check with grapher. It is a horizontal line.

$r_1 = \frac{1}{2\sin(\theta)}$ or r1=1/(2sin(θ))

Graph in a standard window.



b. $r = \frac{\tan\theta}{\cos\theta - \sin\theta}$

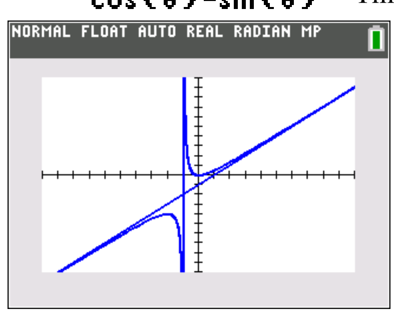
$r\cos\theta - r\sin\theta = \tan\theta$ *Goal:* Create $r\cos\theta$ and $r\sin\theta$ to replace with x and y

$x - y = \frac{y}{x}$ Replace $\tan\theta$ with $\frac{y}{x}$

$$\boxed{x^2 - xy = y}$$

Graph in a standard window. Set θstep = 0.1

$r_1 = \frac{\tan(\theta)}{\cos(\theta)-\sin(\theta)}$ This is a rotated hyperbola.

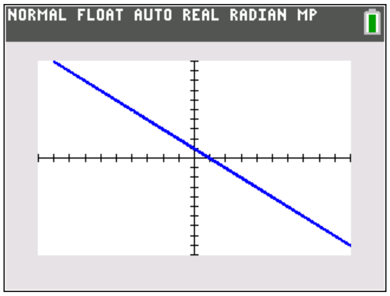


c. $r = \dfrac{1}{\cos\theta + \sin\theta}$

$r\cos\theta + r\sin\theta = 1$ *Goal:* Create $r\cos\theta$ and $r\sin\theta$

$x + y = 1$

Check with grapher. It is the line $\boxed{y = -x + 1}$
Graph in a standard window.



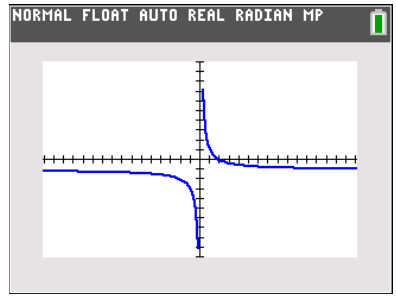
d. $r\cot\theta + r^2\cos\theta = 2\csc\theta$

$r\cos\theta + r^2\cos\theta\sin\theta = 2$
$r\cos\theta + r\cos\theta\, r\sin\theta = 2$
$x + x \cdot y = 2$
$x \cdot y = 2 - x$

$\boxed{y = \dfrac{2-x}{x}}$ or $\boxed{y = \dfrac{2}{x} - 1}$

It is implicit in r so we can't use a polar grapher.
Use function mode and graph in a standard window.

$Y_1 = \dfrac{2-X}{X}$ This is a rational function with vertical asymptote $x = 0$.

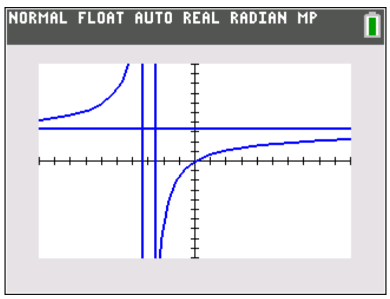


e. $r = 3\csc\theta(1 - \tan\theta)$

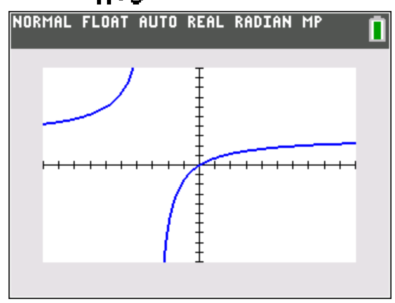
$r = 3 \cdot \dfrac{1}{\sin\theta}(1 - \tan\theta)$
$r\sin\theta = 3(1 - \tan\theta)$
$r\sin\theta = 3 - 3\tan\theta$
$y = 3 - 3\cdot\dfrac{y}{x}$
$x \cdot y = 3x - 3y$
$x\cdot y + 3y = 3x$
$y(x + 3) = 3x$

$\boxed{y = \dfrac{3x}{x+3}}$

$r_1 = \dfrac{3}{\sin(\theta)}(1-\tan(\theta))$



$Y_1 = \dfrac{3X}{X+3}$



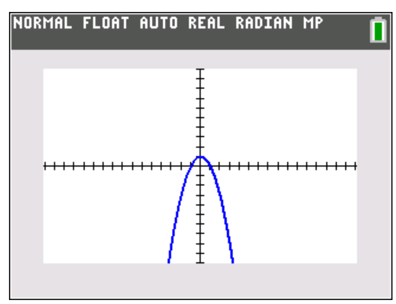
f. $r^2\cos\theta + r\tan\theta = \sec\theta$

$r\cos\theta + \tan\theta = \dfrac{1}{r\cos\theta}$ Multiply all terms by $\dfrac{1}{r}$

$x + \dfrac{y}{x} = \dfrac{1}{x}$

$\dfrac{y}{x} = \dfrac{1}{x} - x$

$\boxed{y = 1 - x^2}$ Multiply all terms by x



g. $r^2 = \sec^2\theta\tan\theta$

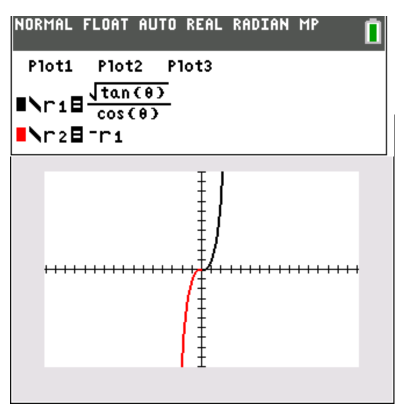
$r^2 = \dfrac{1}{\cos^2\theta}\cdot\dfrac{\sin\theta}{\cos\theta}$ Clear fractions. Make $r\cos\theta$

$r^2\cos^2\theta\cos\theta = \sin\theta$ Multiply all terms by r

$r^2\cos^2\theta\, r\cos\theta = r\sin\theta$ Make $r\cos\theta$ and $r\sin\theta$

$x^2 \cdot x = y$

$\boxed{y = x^3}$



h. $r = \dfrac{2\csc\theta}{\cot\theta + r\cos\theta}$

$r = \dfrac{1}{\sin\theta} \cdot \dfrac{2}{\cot\theta + r\cos\theta}$ Rewrite csc θ as the reciprocal of sin θ.

$1 = \dfrac{1}{r\sin\theta} \cdot \dfrac{2}{\cot\theta + r\cos\theta}$ Divide by 1 / r

$1 = \dfrac{1}{y} \cdot \dfrac{2}{\dfrac{x}{y} + x}$ Replace $r$sin θ with y, $r$cos θ with x, and cot θ with x / y

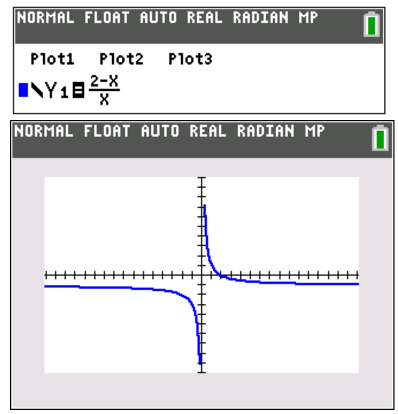
$1 = \dfrac{2}{y \cdot \dfrac{x}{y} + y \cdot x}$ Distribute. One out.

$1 = \dfrac{2}{x + yx}$ Clear fractions

$x + yx = 2$ Isolate the term containing y

$yx = 2 - x$ Divide both sides by x.

$y = \boxed{\dfrac{2-x}{x}}$ or $y = \boxed{\dfrac{2}{x} - 1}$

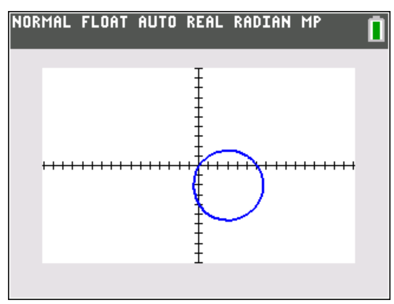


3. Given the Cartesian equation in terms of x and y, write the polar equation in terms of r and θ. Your equation should begin with "r ="

a. $x^2 + y^2 = 6x - 4y$

$r^2 = 6r\cos\theta - 4r\sin\theta$ Divide both sides by r.

$\boxed{r = 6\cos\theta - 4\sin\theta}$

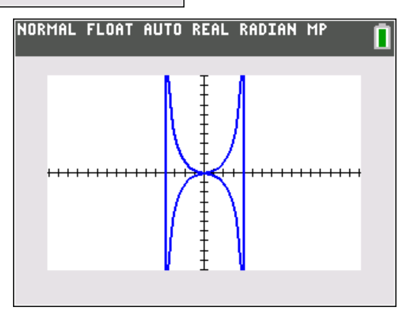


b. $x^2(x^2 + y^2) = 16y^2$

$(x^2 + y^2) = \dfrac{16y^2}{x^2}$

$r^2 = 16\tan^2\theta$ Take square roots of both sides.

$\boxed{r = 4\tan\theta}$



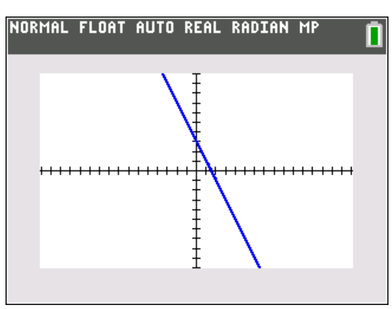
c. $y = 3 - 2x$

$r\sin\theta = 3 - 2r\cos\theta$

$r\sin\theta + 2r\cos\theta = 3$

$r(\sin\theta + 2r\cos\theta) = 3$

$\boxed{r = \dfrac{3}{2\cos\theta + \sin\theta}}$



4. a.

$(x^2+y^2)^2 = 128xy$ Substitute $r^2 = x^2+y^2$, $x = r\cos\theta$, and $y = r\sin\theta$.

$(r^2)^2 = 128(r\cos\theta)(r\sin\theta)$ Factor out r^2. Rewrite $128\cos\theta\sin\theta = 64\cdot(2\cos\theta\sin\theta)$.

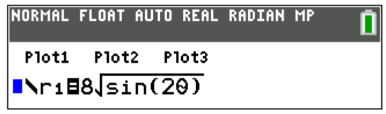
$r^4 = 128(r\cos\theta)(r\sin\theta)$ Rewrite $128\cos\theta\sin\theta = 64\cdot(2\cos\theta\sin\theta)$.

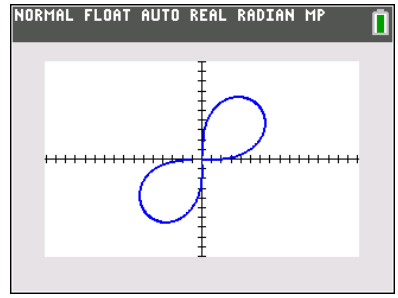
$r^4 = r^2\cdot 64\cdot(2\cos\theta\sin\theta)$ Divide both sides by r^2.

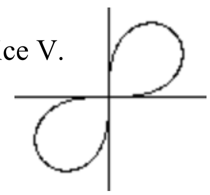
$r^2 = 64\cdot(2\cos\theta\sin\theta)$ Rewrite $2\cos\theta\sin\theta = \sin 2\theta$.

$r^2 = 64\cdot(\sin 2\theta)$ Take square roots of both sides.

$r = 8\sqrt{\sin 2\theta}$





Choice V. 

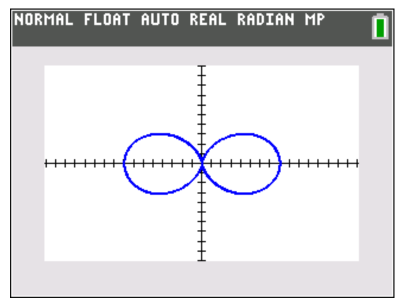
b. $(x^2+y^2)^3 = 64x^4$

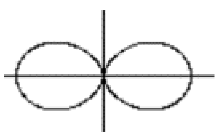
$(r^2)^3 = 64(r\cos\theta)^4$

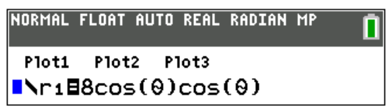
$r^6 = 64r^4\cos^4\theta$ Divide both sides by r^4

$r^2 = 64\cos^4\theta$ Take square roots of both sides.

$r = 8\cos^2\theta$



Choice I. 



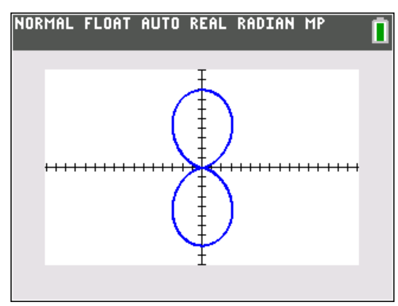
c. $(x^2+y^2)^3 = 64y^4$

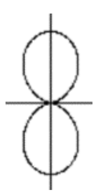
$(r^2)^3 = 64(r\sin\theta)^4$ Substitute $r^2 = x^2+y^2$ and $y = r\sin\theta$. Use laws of exponents to simplify.

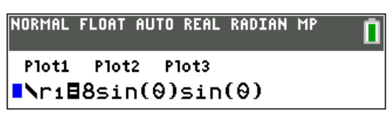
$r^6 = 64r^4\sin^4\theta$ Divide both sides by r^4.

$r^2 = 64\sin^4\theta$ Take square roots of both sides.

$r = 8\sin^2\theta$

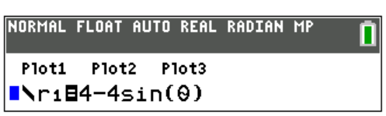


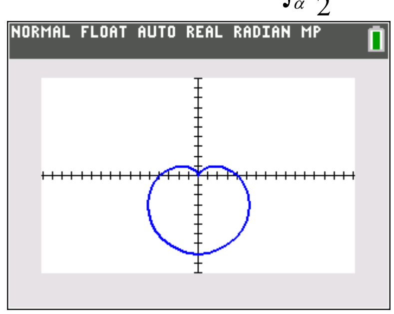
Choice VI. 

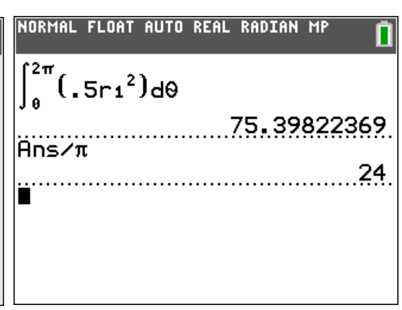


5. a. $r = 4 - 4\sin\theta$

Use the cool fact that the area from $\theta = \alpha$ to $\theta = \beta$ inside a polar graph is $\int_\alpha^\beta \frac{1}{2}r^2\,d\theta$

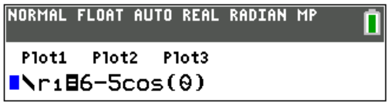


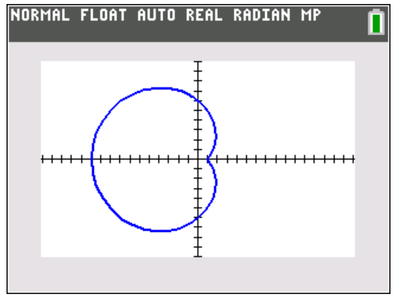


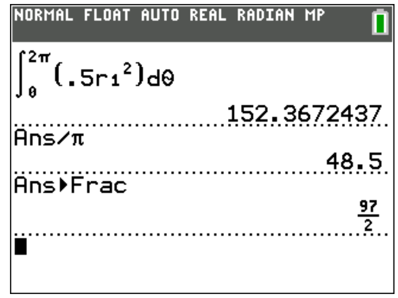


The exact area is **24π**.

b. $r = 6 - 5\cos\theta$

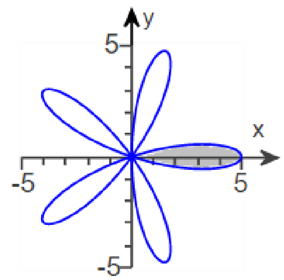






The exact area is **48.5π**.

You could also report $\dfrac{97}{2}\pi$



6. Use the cool fact that the area from $\theta = \alpha$ to $\theta = \beta$ inside a polar graph is $\int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$

a. Find the exact area of the region inside one leaf of the 5-leaved rose $r = 5\cos 5\theta$
You can use the FNINT command, but provide an exact area.

Solve when $r = 5\cos 5\theta = 0$.
The smallest negative value is the solution to

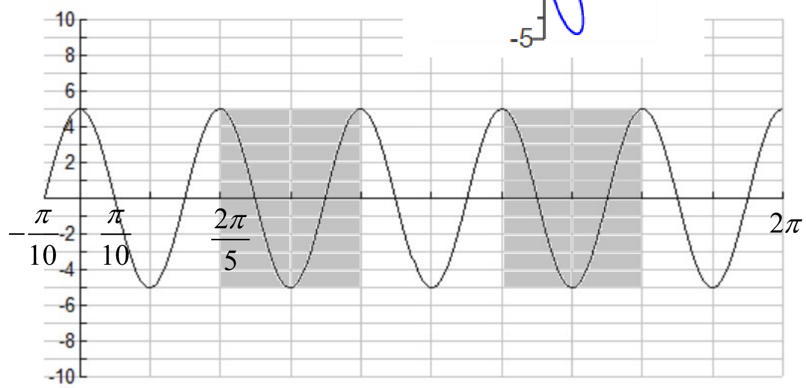
$$5\theta = -\frac{\pi}{2}.$$

Divide both sides by 5: $\theta = -\dfrac{\pi}{10}$

The smallest positive value is the solution to

$$5\theta = \frac{\pi}{2}.$$

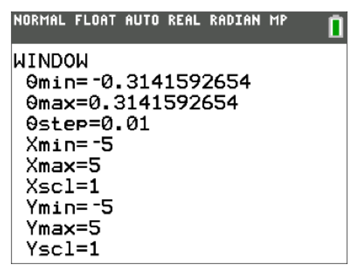
Divide both sides by 5: $\theta = \dfrac{\pi}{10}$



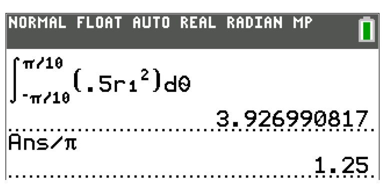
We can also solve this graphically.

Sketch a cosine graph $y = 5\cos 5\theta$. There are 5 cycles in one interval of $[0, 2\pi]$, so a first cycle happens on $[0, \frac{2\pi}{5}]$.

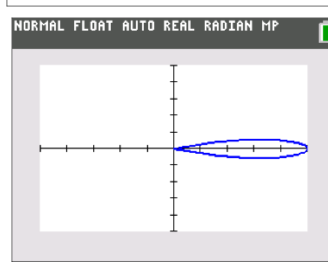
We can sketch one petal in the polar grapher if we set $\theta\text{min} = -\dfrac{\pi}{10}$ and $\theta\text{ max} = \dfrac{\pi}{10}$.



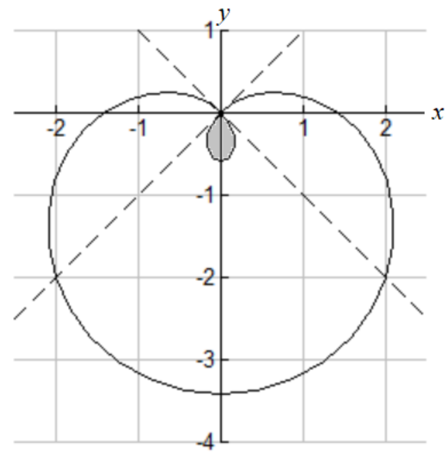
r1■5cos(5θ)



$$\int_{-\pi/10}^{\pi/10} \frac{1}{2}(5\cos 5\theta)^2 \, d\theta = \frac{5}{4}\pi \text{ or } 1.25\pi$$



b. Set up the integral to calculate the area of the region inside the inner loop of the limaçon $r = \sqrt{2} - 2\sin\theta$. Use the FNINT command to find the area and approximate it the area to two decimal places.



To find the integration limits, find where $r = \sqrt{2} - 2\sin\theta = 0$
where $0 \le \theta < 2\pi$, since this will be where the inner loop starts and ends.
TIP: The dashed lines in the above graph are the
polar equations $\theta = \alpha$ and $\theta = \beta$, where α and β are the lower and upper limits of integration. You can enter these values in your polar grapher as *θmin* and *θmax* to check that you have sketched only the inner loop.

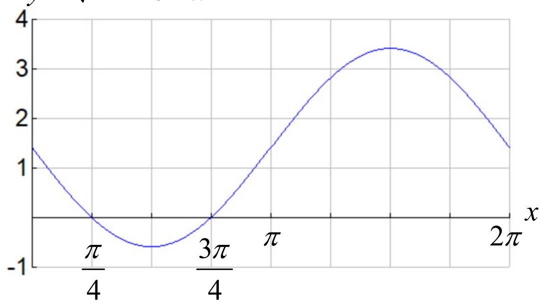
Solve $r = \sqrt{2} - 2\sin\theta = 0$ graphically or algebraically.

$$2\sin\theta = \sqrt{2}$$

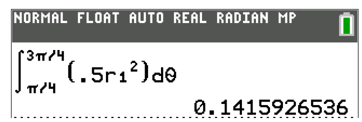
$$\sin\theta = \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

$y = \sqrt{2} - 2\sin x$



$r_1 = \sqrt{2} - 2\sin(\theta)$



$$\int_{\boxed{\pi/4}}^{\boxed{3\pi/4}} \boxed{\frac{1}{2}(\sqrt{2} - 2\sin\theta)^2}\, d\theta \approx \boxed{0.14}$$