

Fun Facts: $\sum_{k=0}^{n-1} a(r)^k = a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$ is the Big Daddy Series. Another Fun Fact: $\lim_{n \rightarrow \infty} (1 + \frac{r}{n})^n = e^r$.

1. For what values of p does $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converge? $p > 1$ For what values of r does $\sum_{k=1}^{\infty} ar^{k-1}$ converge? $|r| < 1$

You could also write $-1 < r < 1$

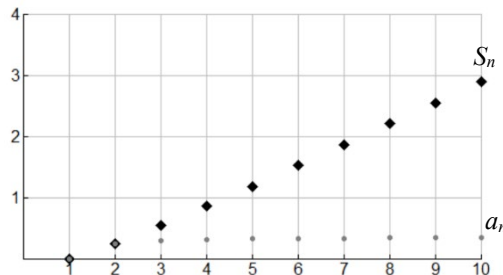
2. Shown to the right is a plot of the terms a_n and the n th partial sums S_n for the series $\sum_{k=1}^{\infty} \left(1 - \frac{1}{k}\right)^k$.

- a. Circle the correct choice: We know that $\lim_{k \rightarrow \infty} \left(1 - \frac{1}{k}\right)^k$ does not equal 0.

This means the n th Term Test for Divergence says that

the series $\sum_{k=1}^{\infty} \left(1 - \frac{1}{k}\right)^k$ diverges.

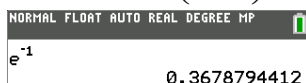
S_n climbs to infinity.



- b. The plot shows $\lim_{k \rightarrow \infty} \left(1 - \frac{1}{k}\right)^k \approx 0.3678794411714423215955237701614608674458111310317678345078368016974614957448998034$

What is the exact value of this number? $\lim_{k \rightarrow \infty} \left(1 - \frac{1}{k}\right)^k =$ e^{-1}

$\lim_{k \rightarrow \infty} \left(1 - \frac{1}{k}\right)^k = \lim_{k \rightarrow \infty} \left(1 + \frac{-1}{k}\right)^k = e^{-1}$
Write in the box an **exact** number or DNE or ∞ or $-\infty$.



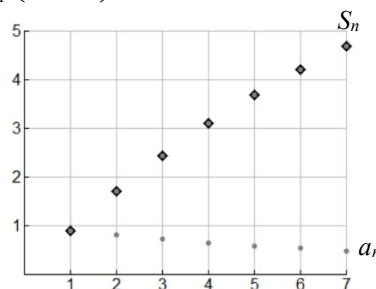
You can check with a grapher.

3. Shown to the right is a plot of the terms a_n and the n th partial sums S_n for the series $\sum_{k=1}^{\infty} \left(1 - \frac{1}{10}\right)^k$.

- a. Circle the correct choice: We know that $\lim_{k \rightarrow \infty} \left(1 - \frac{1}{10}\right)^k$ does not equal 0,

This means by the n th Term Test for Divergence that

the series $\sum_{k=1}^{\infty} \left(1 - \frac{1}{10}\right)^k$ is nonconclusive.



- b. The series $\sum_{k=1}^{\infty} \left(1 - \frac{1}{10}\right)^k =$ 9 Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Geometric with $r = \frac{9}{10}$ and $a = \frac{9}{10}$ so the sum is $\frac{a}{1-r} = \frac{\frac{9}{10}}{1-\frac{9}{10}} = \frac{\frac{9}{10}}{\frac{1}{10}} = 9$

4. Suppose $\sum_{k=1}^{\infty} a_k$ is any series. Circle True or False. If False, give a counterexample which shows it is False. If True, leave blank.

- a. True False If $\lim_{k \rightarrow \infty} a_k = 0$, then $\sum_{k=1}^{\infty} a_k$ converges. **Harmonic Series or any p series with $p \leq 1$**

- b. True False If the limit of the sequence of partial sums S_n exists, i.e., $\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \sum_{k=1}^n a_k = L < \infty$, then $\sum_{k=1}^{\infty} a_k = L$, where L is some finite number. **This is essentially what it means for a series to converge.**

- c. True False If the limit of the sequence a_k exists, i.e., $\lim_{k \rightarrow \infty} a_k = L$, then the series $\sum_{k=1}^{\infty} a_k$ converges.

Use the series in Question 2 or any a_k where $\lim_{k \rightarrow \infty} a_k = L \neq 0$.

- d. True False If $\sum_{k=1}^{\infty} a_k$ converges, then the sequence of partial sums S_n approaches 0, i.e., $\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \sum_{k=1}^n a_k = 0$.

Use the series in Question 3 or any geometric series with $|r| < 1$ and $a \neq 0$.

5. Determine whether $\sum_{n=1}^{\infty} \frac{80+n}{n^2}$ converges or diverges by answering the questions below.

a. Complete. $\sum_{n=1}^{\infty} \frac{80+n}{n^2}$ can be written as the sum of two series;

a p -series with $\frac{80}{n^2}$ and $\frac{1}{n}$.
 $\{p < 2, p > 1\}$ {the harmonic series, another p -series with $p < 1$, another p -series with $p > 1$ }

b. Write $\sum_{n=1}^{\infty} \frac{80+n}{n^2}$ as the sum of two p -series with $p \neq 1$ (or a p -series with $p \neq 1$ and the harmonic series). Simplify.

$$\sum_{n=1}^{\infty} \frac{80+n}{n^2} = \sum_{n=1}^{\infty} \frac{80}{n^2} + \sum_{n=1}^{\infty} \frac{1}{n}$$

converges ($p = 2 > 1$) diverges ($p = 1$)

c. Circle the correct choice and fill in the blanks: The series $\sum_{n=1}^{\infty} \frac{80+n}{n^2}$ will converge.
 $\{converge, diverge\}$

6. Determine whether $\sum_{n=1}^{\infty} \frac{80+n}{n^5}$ converges or diverges by answering the questions below.

a. Complete. $\sum_{n=1}^{\infty} \frac{80+n}{n^5}$ can be written as the sum of two series;

a p -series with $\frac{80}{n^5}$ and $\frac{1}{n^4}$.
 $\{p < 5, p > 1\}$ {the harmonic series, another p -series with $p < 1$, another p -series with $p > 1$ }

b. Write $\sum_{n=1}^{\infty} \frac{80+n}{n^5}$ as the sum of two p -series with $p \neq 1$ (or a p -series with $p \neq 1$ and the harmonic series). Simplify.

$$\sum_{n=1}^{\infty} \frac{80+n}{n^5} = \sum_{n=1}^{\infty} \frac{80}{n^5} + \sum_{n=1}^{\infty} \frac{1}{n^4}$$

converges ($p = 5 > 1$) converges ($p = 4 > 1$)

c. Circle the correct choice and fill in the blanks: The series $\sum_{n=1}^{\infty} \frac{80+n}{n^5}$ will converge.
 $\{converge, diverge\}$

7. Because $f(x) = \frac{80}{1+x^2}$ is decreasing for $x \geq 1$ we can use the Integral Test to show $\sum_{n=1}^{\infty} \frac{80}{1+n^2}$ converges or diverges.

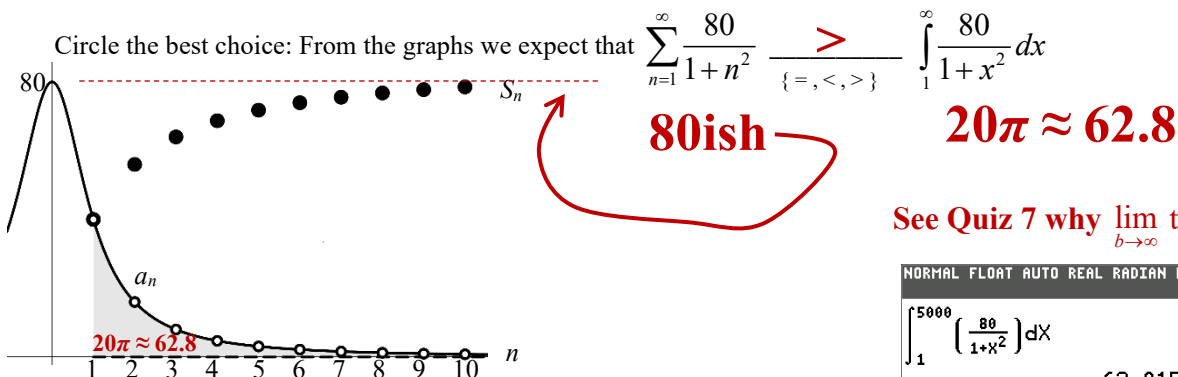
a. Find the expression $\int_1^b \frac{80}{1+x^2} dx$ in terms of b. $\int_1^b \frac{80}{1+x^2} dx = 80 \tan^{-1} b - 20\pi$

$$\int_1^b \frac{80}{1+x^2} dx = 80 \tan^{-1} x \Big|_1^b = 80 \tan^{-1} b - 80 \tan^{-1} 1 = 80 \tan^{-1} b - 80 \cdot \frac{\pi}{4} = 80 \tan^{-1} b - 20\pi$$

b. The series $\sum_{n=1}^{\infty} \frac{80}{1+n^2}$ will converge because $\int_1^{\infty} \frac{80}{1+x^2} dx = 20\pi$. Write in the box an **exact** number or DNE or ∞ or $-\infty$.

$$\int_1^{\infty} \frac{80}{1+x^2} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{80}{1+x^2} dx = \lim_{b \rightarrow \infty} (80 \tan^{-1} b - 20\pi) = 80 \lim_{b \rightarrow \infty} \tan^{-1} b - 20\pi = 80 \cdot \frac{\pi}{2} - 20\pi = 40\pi - 20\pi = 20\pi$$

c. Shown below is a plot of a_n and S_n for the series $\sum_{n=1}^{\infty} \frac{80}{1+n^2}$, as well as the area representing $\int_1^{\infty} \frac{80}{1+x^2} dx$.



8. Consider $\sum_{n=1}^{\infty} \frac{\ln n}{n} = 0 + \frac{\ln 2}{2} + \frac{\ln 3}{3} + \frac{\ln 4}{4} + \dots$ which, for $n > 2$, is a decreasing series of positive terms.

a. Circle the correct choice: We know that $\lim_{n \rightarrow \infty} \frac{\ln n}{n}$ _____ equal 0. This means the n th Term Test for Divergence says that

the series $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ _____.

$\lim_{n \rightarrow \infty} \frac{\ln n}{n}$ is indeterminate ($\frac{\infty}{\infty}$) so we can apply L'Hôpital's Rule: $\lim_{n \rightarrow \infty} \frac{\ln n}{n} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{1} = 0$.

b. Find the expression $\int_1^b \frac{\ln x}{x} dx$ in terms of b .

$$\int_1^b \frac{\ln x}{x} dx = \left[\frac{\ln^2 b}{2} - \frac{\ln^2 1}{2} \right] = \frac{1}{2} \ln^2 b$$

Use u -substitution with

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int \frac{\ln x}{x} dx = \int \ln x \cdot \frac{1}{x} dx = \int u \cdot du = \frac{u^2}{2} + C = \frac{\ln^2 x}{2} + C$$

c. The series $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ will _____ because $\int_1^{\infty} \frac{\ln x}{x} dx = \infty$. Write in the box an **exact** number or DNE or ∞ or $-\infty$.