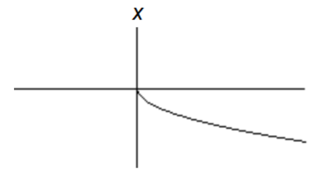
Practice Questions over 12.1

1. Eliminate the parameter, t, to obtain an equation of the form $y = f(x)$.

a. $x = \sqrt[3]{t-1}$, $y = \cos t$ (There is no new domain restriction.)
$t = x^3 + 1$ so the curve is $y = \cos(x^3 + 1)$

b. $x = \sqrt[3]{t-1}$, $y = \cos\sqrt[3]{t-1}$ (There is no new domain restriction.)
Replace $\sqrt[3]{t-1}$ with x so the curve is $y = \cos x$.

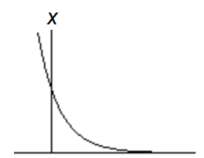
c. $x = -\sqrt{t}$, $y = -3\sqrt{t} + 6e^{\sqrt{t}}$ (Specify the domain restriction.)



Replace $-\sqrt{t}$ with x so the curve is
$y = -3\sqrt{t} + 6e^{\sqrt{t}}$
$y = 3(-\sqrt{t}) + 6e^{-(-\sqrt{t})}$
$y = 3x + 6e^{-x}$.
We have $y = 3x + 6e^{-x}$.
Restrict the domain to $x \le 0$ since the range of the graph of $x = -\sqrt{t}$ is $x \le 0$.

d. $x = -\sqrt{t}$, $y = 2t + 1$ (Specify the domain restriction.)
Square both sides of $x = -\sqrt{t}$ and we have $x^2 = t$, but restrict the domain to $x \le 0$.
Replace t with x^2 so the curve is $y = 2x^2 + 1$, $x \le 0$.

e. $x = e^{-t}$, $y = 7e^{-3t}$ (Specify the domain restriction.)
Replace e^{-t} with x so the curve is $y = 7e^{-3t} = 7(e^{-t})^3 = 7x^3$ with $x > 0$.
Restrict the domain to $x > 0$ since the range of the graph of $x = e^{-t}$ is $x > 0$.



f. $x = 4\sin t$, $y = 3 + 4\cos t$
Use $\cos^2 t + \sin^2 t = 1$ with $\sin t = \dfrac{x}{4}$ and $\cos t = \dfrac{y-3}{4}$.

The curve is $\dfrac{(y-3)^2}{4^2} + \dfrac{x^2}{4^2} = 1$ but we can also write $(y-3)^2 + x^2 = 4^2$.

Or $\dfrac{x^2}{4^2} + \dfrac{(y-3)^2}{4^2} = 1$ or $x^2 + (y-3)^2 = 4^2$. This is a circle with center (0, −3) and radius 4.

g. $x = 3\sin t$, $y = 3 - 6\cos t$
Use $\sin^2 t + \cos^2 t = 1$ with $\sin t = \dfrac{x}{3}$ and $\cos t = \dfrac{y-3}{-6}$.

The curve is $\dfrac{x^2}{3^2} + \left(\dfrac{y-3}{-6}\right)^2 = 1$ or $\dfrac{x^2}{3^2} + \dfrac{(y-3)^2}{6^2} = 1$

h. $x = 3\cos t$, $y = 3 - 9\cos^2 t$
Replace $\cos t$ with $\dfrac{x}{3}$ and $\cos^2 t$ with $\dfrac{x^2}{9}$ so the curve is $y = 3 - 9\cdot\dfrac{x^2}{9} = 3 - x^2$, $-3 \le x \le 3$.

2. Write a set of parametric equations $x = f(t)$, $y = g(t)$ for the curve

a. $x = 4y^5 - 3y^2 + 2\cos y - e^{7y}$

$x = 4t^5 - 3t^2 + 2\cos t - e^{7t}$ and $y = t$

b. The circle $(x-1)^2 + (y+2)^2 = 49$

The center is $(1, -2)$ and radius is 7.

Parametric equations: $x = 7\cos t + 1$

$y = 7\sin t - 2$

Other options are possible.

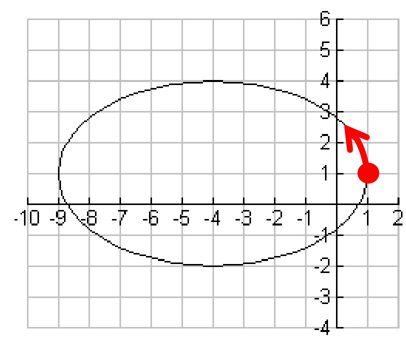
c. The ellipse $\dfrac{(x-5)^2}{9} + \dfrac{(y+2)^2}{4} = 1$

The center is $(5, -2)$, RUN = 3, RISE = 2.

Parametric equations: $x = 3\cos t + 5$

$y = 2\sin t - 2$

The vertices are (1, 1) and (−9, 1).



d. The ellipse shown with the initial value of $t = 0$, $x = 1$, $y = 1$ traveling counterclockwise. Report the center, RUN, RISE, and vertices. Also report implicit form. Center is $(-4, 1)$.

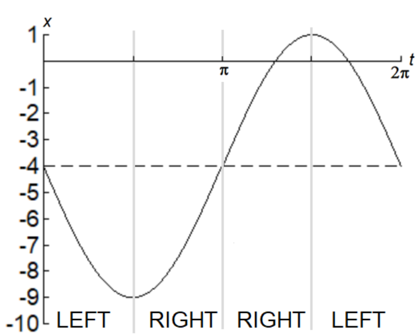
RUN = 5, RISE = 3.

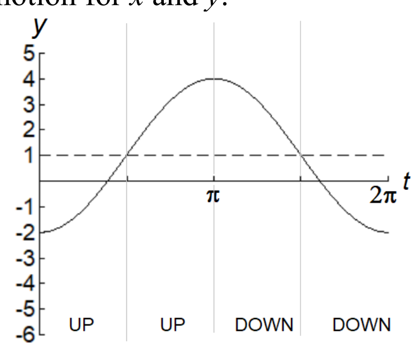
Parametric equations: $x = 5\cos t - 4$

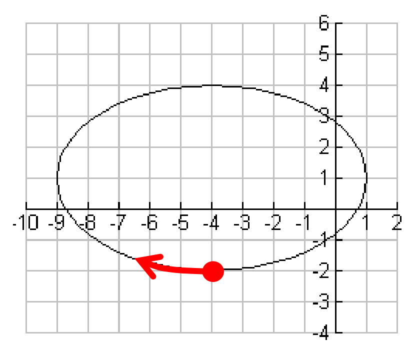
$y = 3\sin t + 1$

The implicit form of the ellipse is $\dfrac{(x+4)^2}{25} + \dfrac{(y-1)^2}{9} = 1$

e. The same ellipse shown to the right with the initial value of $t = 0$, $x = -4$, $y = -2$ traveling clockwise. This means we have the following motion for x and y:



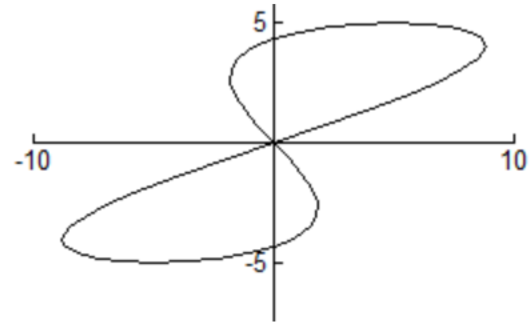




Parametric equations: $x = -5\sin t - 4$

$y = -3\cos t + 1$

3. The graph of the parametric equations $x = 5\sin t - 5\sin 2t$ and $y = 5\sin t$ is shown for $0 \le t \le 2\pi$.



$x = 5\sin t - 5\sin 2t$

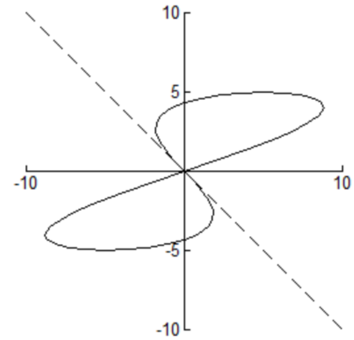
$$\frac{dx}{dt} = 5\cos t - 5(\cos 2t)\cdot\frac{d}{dt}(2t)$$

$$= 5\cos t - 10\cos 2t$$

$y = 5\sin t$

$$\frac{dy}{dt} = 5\cos t$$

a. Evaluate $\frac{dy}{dx}$ at the origin when $t = 0$. $\frac{dy}{dx} = \boxed{-1}$

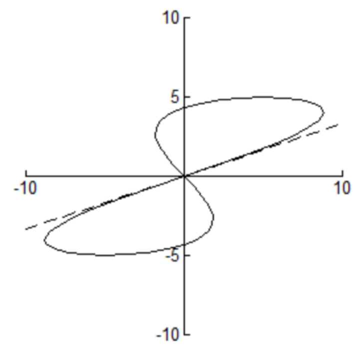


If $t = 0$, then $\frac{dx}{dt} = 5\cos(t) - 10\cos(2t) = 5\cos(0) - 10\cos(0) = 5(1) - 10(1) = -5$

If $t = 0$, then $\frac{dy}{dt} = 5\cos(0) = 5$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{5}{-5} = -1$$

b. Evaluate $\frac{dy}{dx}$ at the origin when $t = \pi$. $\frac{dy}{dx} = \boxed{\frac{1}{3}}$



If $t = \pi$, then $\frac{dx}{dt} = 5\cos(t) - 10\cos(2t) = 5\cos(\pi) - 10\cos(2\pi) = -5 - 10 = -15$

If $t = \pi$, then $\frac{dy}{dt} = 5\cos(\pi) = -5$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-5}{-15} = \frac{1}{3}$$

c. The arc length from $t = 0$ to $t = 2\pi$ of this curve is given by $\int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$.

Complete the boxes to set up the integral to find the arc length. You need not simplify. Then use FNINT to find the arc length rounded to the nearest whole number.

$$\int_0^{2\pi} \sqrt{\boxed{(5\cos t - 10\cos 2t)^2 + (5\cos t)^2}}\,dt \approx \boxed{50}$$