MA 16600 QUIZ 13A (HW 30-32) KEY

(2) **1.** A. Replace the rectangular equation $x = 5$ with an appropriate polar equation. Show work please.

$$r\cos\theta = 5$$
$$r = \frac{5}{\cos\theta}$$
$$r = 5\sec\theta$$

B. Replace the rectangular equation $x = 7$ with an appropriate polar equation. Show work please.

$$r\cos\theta = 7$$
$$r = \frac{7}{\cos\theta}$$
$$r = 7\sec\theta$$

(3) **2.** A. Replace the polar equation $r = 5\cos\theta$ with an appropriate rectangular equation. Show work please.

The implicit form of the equation is fine.
In rectangular, the equation is $x^2 + y^2 = 5x$

$$r = 5\cos\theta$$
$$r^2 = 5r\cos\theta$$
$$x^2 + y^2 = 5x$$

B. Replace the polar equation $r = 5\cos\theta$ with an appropriate rectangular equation. Show work please.

The implicit form of the equation is fine.
In rectangular, the equation is $x^2 + y^2 = 7y$

$$r = 7\sin\theta$$
$$r^2 = 7r\sin\theta$$
$$x^2 + y^2 = 7y$$

(4) **3.** Replace the polar equation with an appropriate rectangular equation. Show work please.

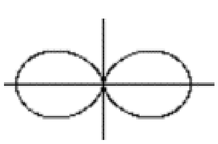
$$r = 3\sec\theta(1-\tan\theta)$$
$$r = \frac{3}{\cos\theta}(1-\tan\theta)$$
$$r\cos\theta = 3(1-\tan\theta)$$
$$x = 3(1-\frac{y}{x})$$
$$\frac{x}{3} = 1-\frac{y}{x}$$
$$x\cdot\frac{x}{3} = x\cdot(1-\frac{y}{x})$$
$$\frac{x^2}{3} = x - y$$
$$y = x - \frac{x^2}{3}$$

$$r = 2\sec\theta(1+\tan\theta)$$
$$r = \frac{2}{\cos\theta}(1+\tan\theta)$$
$$r\cos\theta = 2(1+\tan\theta)$$
$$x = 2(1+\frac{y}{x})$$
$$\frac{x}{2} = 1+\frac{y}{x}$$
$$x\cdot\frac{x}{2} = x\cdot(1+\frac{y}{x})$$
$$\frac{x^2}{2} = x + y$$
$$y = \frac{x^2}{2} - x$$

Any equations that are equivalent to this are acceptable.

4. Sketch a graph of the curve described by $(x^2 + y^2)^3 = 16x^4$ by first replacing it with a polar equation and simplifying. Show work please.

(4) **a**. In polar, the equation is: $r = 4\cos^2\theta$

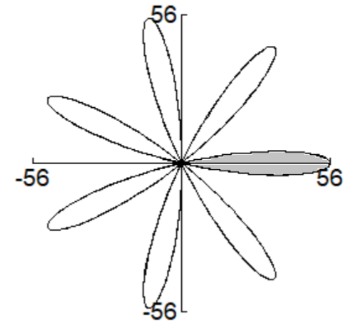
b. 

$$\left(x^2+y^2\right)^3 = 16x^4$$
$$\left(r^2\right)^3 = 16r^4\cos^4\theta$$
$$r^2r^2r^2 = 16r^4\cos^4\theta$$
$$r^2 = 16\cos^4\theta$$
$$r = 4\cos^2\theta$$

(4) **5.** The area from $\theta = \alpha$ to $\theta = \beta$ inside a polar graph is $\int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta$

A. Find the exact area of the region inside one petal of the rose. You can use the FNINT command, but provide an exact area as a multiple of π.

A. $r = 56\cos 7\theta$



To find when $r = 56\cos 7\theta = 0$, set $7\theta = \pm\frac{\pi}{2}$. We have $\theta = \pm\frac{\pi}{14}$

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r1=56cos(7θ)
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$$\int_{-\pi/14}^{\pi/14} \frac{1}{2}(56\cos 7\theta)^2 \, d\theta = 112\pi$$

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∫(π/14, -π/14) (.5r1²)dθ
                    351.8583772
Ans/π
                            112
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B. $r = 72\cos 9\theta$

To find when $r = 72\cos 9\theta = 0$, set $9\theta = \pm\frac{\pi}{2}$. We have $\theta = \pm\frac{\pi}{18}$

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r1=72cos(9θ)
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$$\int_{-\pi/18}^{\pi/18} \frac{1}{2}(72\cos 9\theta)^2 \, d\theta = 144\pi$$

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∫(π/18, -π/18) (.5r1²)dθ
                    452.3893421
Ans/π
                            144
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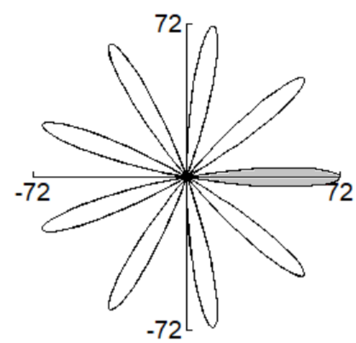
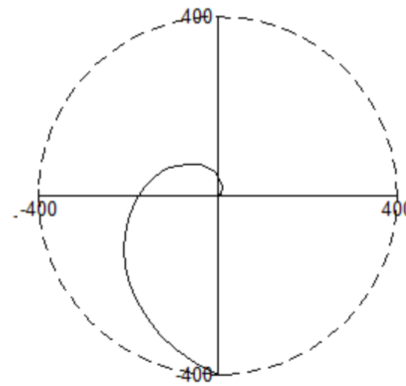


(6) **6.** The arc length from $\theta = 0$ to $\theta = \sqrt{21}$ of a polar spiral is given by $\int_0^{\sqrt{21}} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$.

Report the arc length correct. You can use the FNINT command.

A. $r = 18\theta^2$

$$\int_0^{\sqrt{21}} \sqrt{\boxed{(18\theta^2)^2 + (36\theta)^2}}\, d\theta = \boxed{702}$$



$r_1 \blacksquare 18\theta^2$

$$\int_0^{\sqrt{21}} \left(\sqrt{r_1^2 + (36\theta)^2}\right) d\theta$$

702

B. $r = 30\theta^2$

$$\int_0^{\sqrt{21}} \sqrt{\boxed{(30\theta^2)^2 + (60\theta)^2}}\, d\theta = \boxed{1170}$$

$r_1 \blacksquare 30\theta^2$

$$\int_0^{\sqrt{21}} \left(\sqrt{r_1^2 + (60\theta)^2}\right) d\theta$$

1170

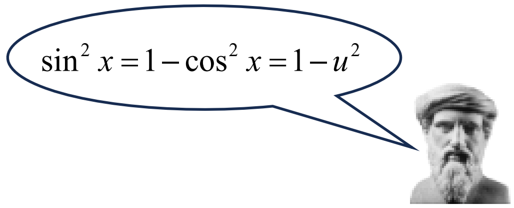
Bonus:

$$\int \sin^3 x \cos^8 x dx$$

Let $u = \cos x$. Then $du = -\sin x\, dx$

$$\int \sin^3 x \cos^8 x dx = \int \sin^2 x \cos^8 x \sin x dx$$

$$= -\int (\sin^2 x) \cdot \cos^8 x \cdot -\sin x dx$$



$$= -\int (1-u^2) \cdot \; u^8 \; \cdot \quad du$$

$$= \int (u^2 - 1) u^8 du$$

$$= \int (u^{10} - u^8) du$$

$$= \frac{u^{11}}{11} - \frac{u^9}{9} + C$$

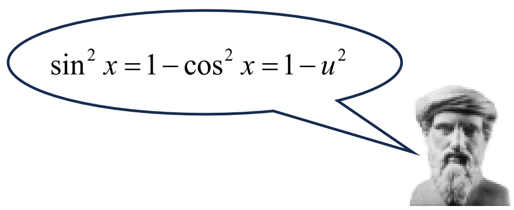
$$= \frac{\cos^{11} x}{11} - \frac{\cos^9 x}{9} + C$$

$$\int \sin^3 x \cos^{10} x dx$$

Let $u = \cos x$. Then $du = -\sin x\, dx$

$$\int \sin^3 x \cos^{10} x dx = \int \sin^2 x \cos^{10} x \sin x dx$$

$$= -\int (\sin^2 x) \cdot \cos^{10} x \cdot -\sin x dx$$



$$= -\int (1-u^2) \cdot \; u^{10} \; \cdot \quad du$$

$$= \int (u^2 - 1) u^{10} du$$

$$= \int (u^{12} - u^{10}) du$$

$$= \frac{u^{13}}{13} - \frac{u^{11}}{11} + C$$

$$= \frac{\cos^{13} x}{13} - \frac{\cos^{11} x}{11} + C$$

$$\int \frac{\cos\theta}{\sin^2\theta} d\theta = ?$$

Method 1: $\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta} = \int \cot\theta \csc\theta\, d\theta = -\csc\theta + C$ (See formula sheet if you forgot.)

Method 2: $\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \frac{1}{\sin^2\theta} \cos\theta\, d\theta$ Let $u = \sin\theta$. Then $du = \cos\theta\, d\theta$

$$= \int \frac{1}{u^2} du = \int u^{-2} du = \frac{u^{-1}}{-1} + C = -\frac{1}{u} + C = -\frac{1}{\sin\theta} + C = -\csc\theta + C$$