

What Did the Girl Rock Say to the Boy Rock?

1. $(x^2 + y^2)^3 = 256y^4$ Substitute $r^2 = x^2 + y^2$ and $y = r \sin \theta$.

$$(r^2)^3 = 256(r \sin \theta)^4$$

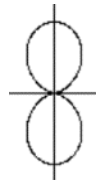
$$r^6 = 256r^4 \sin^4 \theta$$

Divide both sides by r^4 .

$$r^2 = 256 \sin^4 \theta$$

Take square roots of both sides.

$$r = 16 \sin^2 \theta$$



2. $(x^2 + y^2)^3 = 64x^2y^2$ Substitute $r^2 = x^2 + y^2$, $x = r \cos \theta$, and $y = r \sin \theta$.

$$(r^2)^3 = 64(r \cos \theta)^2 (r \sin \theta)^2$$

$$r^6 = 16 \cdot r^4 \cdot 4(\cos \theta \sin \theta)^2$$

Divide both sides by r^4 .

$$r^2 = 16 \cdot 4(\cos \theta \sin \theta)^2$$

Rewrite $4(\cos \theta \sin \theta)^2 = (2 \cos \theta \sin \theta)^2$.

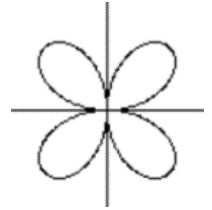
$$r^2 = 16(2 \cos \theta \sin \theta)^2$$

Rewrite $2 \cos \theta \sin \theta = \sin 2\theta$.

$$r^2 = 16(\sin 2\theta)^2$$

Take square roots of both sides.

$$r = 4 \sin 2\theta$$



3. $(x^2 + y^2)^5 = (32xy(x^2 - y^2))^2$ Take square roots of both sides.

$$(x^2 + y^2)^{5/2} = 32xy(x^2 - y^2)$$

Substitute $r^2 = x^2 + y^2$, $x = r \cos \theta$, and $y = r \sin \theta$.

$$(r^2)^{5/2} = 32(r \cos \theta)(r \sin \theta)((r \cos \theta)^2 - (r \sin \theta)^2)$$

Replace $(r^2)^{5/2}$ with r^5 .

$$r^5 = 32(r \cos \theta)(r \sin \theta)((r \cos \theta)^2 - (r \sin \theta)^2)$$

Factor out r^2 .

$$r^5 = 32r^2 \cos \theta \sin \theta r^2 (\cos^2 \theta - \sin^2 \theta)$$

Rewrite $\cos^2 \theta - \sin^2 \theta = \cos 2\theta$.

$$r^5 = 32r^4 \cos \theta \sin \theta (\cos 2\theta)$$

Divide both sides by r^4 .

$$r = 32 \cos \theta \sin \theta (\cos 2\theta)$$

Rewrite $32 \cos \theta \sin \theta = 16 \cdot (2 \cos \theta \sin \theta)$.

$$r = 16 \cdot (2 \cos \theta \sin \theta) (\cos 2\theta)$$

Rewrite $2 \cos \theta \sin \theta = \sin 2\theta$.

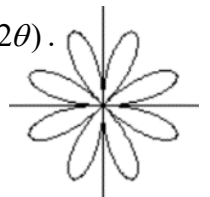
$$r = 16 \cdot (\sin 2\theta) (\cos 2\theta)$$

Rewrite $16 \cos 2\theta \sin 2\theta = 8 \cdot (2 \cos 2\theta \sin 2\theta)$.

$$r = 8 \cdot (2 \cos 2\theta \sin 2\theta)$$

Rewrite $2 \cos 2\theta \sin 2\theta = \sin 4\theta$.

$$r = 8 \sin 4\theta$$



4. $(x^2 + y^2)^2 = 64(x^2 - y^2)$ Substitute $r^2 = x^2 + y^2$, $x = r \cos \theta$, and $y = r \sin \theta$.

$$(r^2)^2 = 64((r \cos \theta)^2 - (r \sin \theta)^2)$$

Factor out r^2 .

$$r^4 = 64r^2 (\cos^2 \theta - \sin^2 \theta)$$

Divide both sides by r^2 .

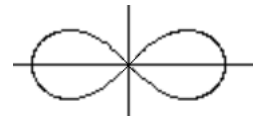
$$r^2 = 64(\cos^2 \theta - \sin^2 \theta)$$

Rewrite $\cos^2 \theta - \sin^2 \theta = \cos 2\theta$.

$$r^2 = 64 \cos 2\theta$$

Take square roots of both sides.

$$r = 8\sqrt{\cos 2\theta}$$



5. $(x^2 + y^2)^3 = 64x^4$ Substitute $r^2 = x^2 + y^2$ and $x = r \cos \theta$

$$(r^2)^3 = 64(r \cos \theta)^4$$

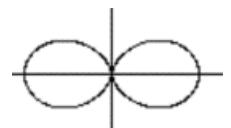
$$r^6 = 64r^4 \cos^4 \theta$$

Divide both sides by r^4 .

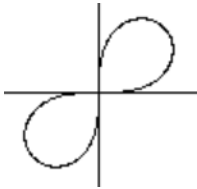
$$r^2 = 64 \cos^4 \theta$$

Take square roots of both sides.

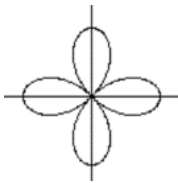
$$r = 8 \cos^2 \theta$$



$$\begin{aligned}
6. \quad (x^2 + y^2)^2 &= 128xy && \text{Substitute } r^2 = x^2 + y^2, \quad x = r \cos \theta, \quad \text{and } y = r \sin \theta. \\
(r^2)^2 &= 128(r \cos \theta)(r \sin \theta) && \text{Factor out } r^2. \text{ Rewrite } 128 \cos \theta \sin \theta = 64 \cdot (2 \cos \theta \sin \theta). \\
r^4 &= 128(r \cos \theta)(r \sin \theta) && \text{Rewrite } 128 \cos \theta \sin \theta = 64 \cdot (2 \cos \theta \sin \theta). \\
r^4 &= r^2 \cdot 64 \cdot (2 \cos \theta \sin \theta) && \text{Divide both sides by } r^2. \\
r^2 &= 64 \cdot (2 \cos \theta \sin \theta) && \text{Rewrite } 2 \cos \theta \sin \theta = \sin 2\theta. \\
r^2 &= 64 \cdot (\sin 2\theta) && \text{Take square roots of both sides.} \\
r &= 8\sqrt{\sin 2\theta}
\end{aligned}$$



$$\begin{aligned}
7. \quad (x^2 + y^2)^3 &= (8(x^2 - y^2))^2 && \text{Take square roots of both sides.} \\
(x^2 + y^2)^{3/2} &= 8(x^2 - y^2) && \text{Substitute } r^2 = x^2 + y^2, \quad x = r \cos \theta, \quad \text{and } y = r \sin \theta. \\
(r^2)^{3/2} &= 8((r \cos \theta)^2 - (r \sin \theta)^2) && \text{Write } (r^2)^{3/2} = r^3. \\
r^3 &= 8(r^2 \cos^2 \theta - r^2 \sin^2 \theta) && \text{Factor out } r^2. \\
r^3 &= 8r^2(\cos^2 \theta - \sin^2 \theta) && \text{Divide both sides by } r^2. \\
r &= 8(\cos^2 \theta - \sin^2 \theta) && \text{Rewrite } \cos^2 \theta - \sin^2 \theta = \cos 2\theta. \\
r &= 8(\cos 2\theta)
\end{aligned}$$



$$\begin{aligned}
8. \quad (x^2 + y^2)^5 &= 64((x^2 - y^2)^2 - 4x^2y^2)^2 && \text{Take square roots of both sides.} \\
(x^2 + y^2)^{5/2} &= 8((x^2 - y^2)^2 - 4x^2y^2) && \text{Substitute } r^2 = x^2 + y^2, \quad x = r \cos \theta, \quad \text{and } y = r \sin \theta. \\
(r^2)^{5/2} &= 8((r^2 \cos^2 \theta - r^2 \sin^2 \theta)^2 - 4r^2 \cos^2 \theta r^2 \sin^2 \theta) && \text{Replace } (r^2)^{5/2} \text{ with } r^5. \text{ Factor out } r^4. \\
r^5 &= 8r^4((\cos^2 \theta - \sin^2 \theta)^2 - 4 \cos^2 \theta \sin^2 \theta) && \text{Divide both sides by } r^4. \\
r &= 8((\cos^2 \theta - \sin^2 \theta)^2 - 4 \cos^2 \theta \sin^2 \theta) \\
r &= 8((\cos^2 \theta - \sin^2 \theta)^2 - 4 \cos^2 \theta \sin^2 \theta) && \text{Rewrite } \cos^2 \theta - \sin^2 \theta = \cos 2\theta. \\
r &= 64(\cos 2\theta)^2 - 4 \cos^2 \theta \sin^2 \theta && \text{Rewrite } 4 \cos^2 \theta \sin^2 \theta = (2 \cos \theta \sin \theta)^2 \\
r &= 8((\cos 2\theta)^2 - (2 \cos \theta \sin \theta)^2) && \text{Rewrite } (2 \cos \theta \sin \theta)^2 = (\sin 2\theta)^2 \\
r &= 8((\cos 2\theta)^2 - (\sin 2\theta)^2) && \text{Factor out 8.} \\
r &= 8(\cos^2 2\theta - \sin^2 2\theta) && \text{Rewrite } \cos^2 2\theta - \sin^2 2\theta = \cos 4\theta. \\
r &= 8(\cos 4\theta)
\end{aligned}$$

