U Got This KEY

Make the nasty integrand pretty. Then report the result in terms of x.

1. $\int x(x-5)^{10}dx$ $\qquad\qquad$ $\int f(u)du = \boxed{\int (u^{11}+5u^{10})du}$

$u = x-5$

$x = u+5$

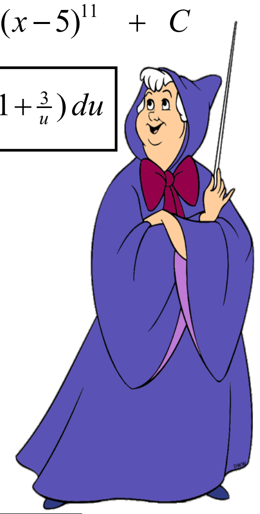
$dx = du$

$$\int x(x-5)^{10}\,dx = \int (u+5)(u)^{10}\,du = \int (u^{11}+5u^{10})du \qquad \text{Distribute}$$

$$= \int u^{11}du + 5\int u^{10}du \qquad \text{Separate, Integrate, Substitute}$$

$$= \frac{u^{12}}{12} + \frac{5u^{11}}{11} + C = \frac{1}{12}(x-5)^{12} + \frac{5}{11}(x-5)^{11} + C$$

2. $\int \frac{x}{x-3}dx$, when written in terms of u, can be much less nasty as the integral $\boxed{\int (1+\frac{3}{u})\,du}$



$u = x-3$

$x = u+3$

$dx = du$

$$\int \frac{x}{x-3}\,dx = \int \frac{u+3}{u}\,du = \int (\tfrac{u}{u}+\tfrac{3}{u})\,du$$

$$= \int (1+\tfrac{3}{u})\,du \qquad \text{Simplify}$$

$$= \int du + 3\int \tfrac{1}{u}\,du \qquad \text{Separate, Integrate, Substitute}$$

$$= u + 3\ln|u| + C = x-3+3\ln|x-3| + C$$

3. $\int \frac{x}{\sqrt{x-100}}dx$, when written in terms of u, can be much less nasty as $\boxed{\int (2u^2+200)\,du}$

One way: $u = \sqrt{x-100}$

$x - 100 = u^2$

$x = u^2 + 100$

$dx = 2udu$

$$\int \frac{x}{\sqrt{x-100}}\,dx = \int \frac{u^2+100}{u}\,2udu = \int \left(\frac{u^2+100}{\cancel{u}}\right)2\cancel{u}du$$

$$= \int 2(u^2+100)\,du \qquad \text{1 out}$$

$$= \int (2u^2+200)\,du \qquad \text{Distribute}$$

$$= 2\int u^2\,du + 200\int u\,du \qquad \text{Separate}$$

$$= \frac{2u^3}{3} + 200u + C \qquad \text{Integrate}$$

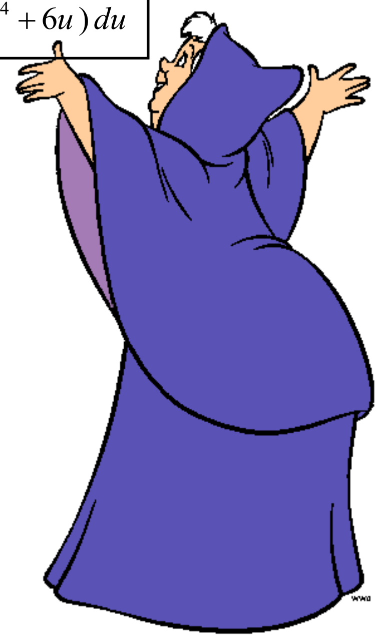
Since $u = \sqrt{x-100} = (x-100)^{1/2}$, then we have $\frac{2}{3}(x-100)^{3/2} + 200(x-100)^{1/2} + C$

Another way: Let $u = x-100$ but this leaves you with

$$\int \frac{x}{\sqrt{x-100}}\,dx = \int \frac{u+100}{\sqrt{u}}\,dx = \int\left(\frac{u}{\sqrt{u}}+\frac{100}{\sqrt{u}}\right)du = \int\left(\frac{u}{u^{1/2}}+\frac{100}{u^{1/2}}\right)du = \int\left(u^{1/2}+100u^{-1/2}\right)du$$

Image sources from Disney Clips (Cinderella's Fairy Godmother and Lady Tremaine)

4. $\int \frac{x}{\sqrt[3]{x-2}}\,dx$, when written in terms of u, can be much less nasty as $\boxed{\int (3u^4 + 6u)\,du}$



One way: $u = \sqrt[3]{x-2}$

$$x - 2 = u^3$$
$$x = u^3 + 2$$
$$dx = \mathbf{3u^2 du}$$

$$\int \frac{x}{\sqrt[3]{x-2}}\,\mathbf{dx} = \int \frac{u^3+2}{u}\,\mathbf{3u^2du} = \int \left(\frac{u^3+2}{\cancel{u}}\right) 3\cancel{u}\cdot u\,du$$

$= \int 3u(u^3+2)\,du$ 1 out

$= \int (3u^4 + 6u)\,du$ Distribute

$= 3\int u^4\,du + 6\int u\,du$ Separate

$= \frac{3u^5}{5} + \frac{6u^2}{2} + C$ Integrate

$= \frac{3}{5}u^5 + 3u^2 + C$

Since $u = \sqrt[3]{x-2} = (x-2)^{1/3}$, then we have $\frac{3}{5}(x-2)^{5/3} + 3(x-2)^{2/3} + C$

5. $\int \sqrt[5]{(3x+2)}(3x+2)dx$ can be written much less nastier as $\boxed{\frac{5}{3}\int u^{10}\,du}$

One way: $u = \sqrt[5]{3x+2}$

$3x + 2 = u^5$ Take differentials of both sides.

$3dx = \mathbf{5u^4du}$ Solve for dx

$dx = \frac{5}{3}\mathbf{u^4du}$

$$\int \sqrt[5]{(3x+2)}\cdot(3x+2)\,\mathbf{(dx)} = \int u\cdot(u)^5\cdot\left(\tfrac{5}{3}\mathbf{u^4du}\right) = \tfrac{5}{3}\int u^{10}\,du$$
$$= \frac{5}{3}\cdot\frac{u^{11}}{11} + C$$
$$= \tfrac{5}{33}u^{11} + C$$

Since $u = \sqrt[5]{3x+2} = (3x+2)^{1/5}$, then we have $\frac{5}{33}(3x+2)^{11/5} + C$

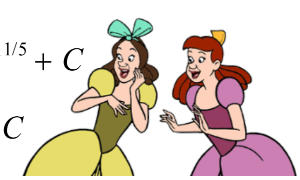
Another way: $u = 3x + 2$

$\mathbf{du} = 3dx$ Solve for dx

$dx = \frac{1}{3}\mathbf{du}$

Then $\int \sqrt[5]{(3x+2)}(3x+2)dx = \int \sqrt[5]{u}\cdot(u)\cdot\left(\frac{1}{3}\mathbf{du}\right) = \frac{1}{3}\int u^{1/5}\cdot(u)\,du = \frac{1}{3}\int u^{6/5}\,du = \frac{1}{3}\cdot\frac{5}{11}u^{11/5} + C$

$= \frac{5}{33}(3x+2)^{11/5} + C$



6. $\int 49x(7x+25)^8\,dx$ can be written as $\boxed{\int (u^9 - 25u^8)\,du}$

$u = 7x + 25$

$du = 7dx$

$$\int 49x(7x+25)^8\,dx = \int 7x(7x+25)^8\cdot\underline{7dx} = \int (u-25)(u)^8\cdot\underline{du} = \int (u^9 - 25u^8)\,du = \tfrac{1}{10}u^{10} - \tfrac{25}{9}u^9 + C$$

Substitute to get $\frac{1}{10}(7x+25)^{10} - \frac{25}{9}(7x+25)^9 + C$