

Practice Questions from HW 26

1. Consider the function $f(x) = \sum_{n=0}^{\infty} 30 \left(\frac{x-40}{20} \right)^n = 30 + 30 \left(\frac{x-40}{20} \right) + 30 \left(\frac{x-40}{20} \right)^2 + 30 \left(\frac{x-40}{20} \right)^3 + \dots$

a. Evaluate. No work need be shown.

Write in the box an exact number or DNE or ∞ or $-\infty$.

$f(72) = \infty$ $f(56) = 150$ $f(20) = \text{DNE}$ $f(60) = \infty$
Handwritten notes: $\sum_{n=0}^{\infty} 30(1.6)^n = \infty$ (so diverges), $\sum_{n=0}^{\infty} 30(.8)^n$ $a=30, r=.8$, $\sum 30(-1)^n = 30-30+30-30+\dots$ diverges by oscillation, $\sum 30(1)^n = 30+30+\dots$

b. For what values of x does $f(x)$ converge? Show work.

$20 < x < 60$ $-1 < \frac{x-40}{20} < 1$

c. Report the sum of the series on its interval of convergence.

$a=30, r = \frac{x-40}{20}$ $\frac{a}{1-r} = \frac{30}{1 - \frac{x-40}{20}} = \frac{600}{20 - (x-40)} = \frac{600}{60-x}$

d. What is true about the graph of $f(x)$ at the left endpoint?

At $x=20$, since $f(20) = \text{DNE}$ we have a (hole, vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point)

e. What is true about the graph of $f(x)$ at the right endpoint?

At $x=60$, since $f(60) = \infty$, we have a (hole, vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point)

2. The series $c(x) = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot 2x^{5k}}{1024^k}$ is a child of the geometric series $\sum_{k=0}^{\infty} ar^k$ where the value of $a = 2$ and $r = \frac{-x^5}{1024}$

which converges for $-4 < x < 4$. You can solve the inequality graphically or with a table.

a. At the left endpoint, $c(x)$ becomes the series $\sum_{k=0}^{\infty} 2 \cdot 1^k = 2 + 2 + 2 + 2 + \dots$ which will (converge, diverge)

What is true about the limit of partial sums S_n ? $\lim_{n \rightarrow \infty} S_n = \infty$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $c(x)$ at the left endpoint?

(hole, vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point)

b. At the right endpoint, $c(x)$ becomes the series $\sum_{k=0}^{\infty} 2 \cdot (-1)^k = 2 + (-2) + 2 + (-2) + \dots$ which will (converge, diverge)

What is true about the limit of partial sums S_n ? $\lim_{n \rightarrow \infty} S_n = \text{DNE}$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $c(x)$ at the right endpoint?

(hole, vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point)

c. Report the sum of the series on its interval of convergence.

$a=2, r = \frac{-x^5}{1024}$ $\frac{a}{1-r} = \frac{2}{1 - \frac{-x^5}{1024}} = \frac{2048}{1024 + x^5}$

3. Complete: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots = \ln 2$. The name of this series is called the Alternating Harmonic series.

Write in the box an exact number or DNE or ∞ or $-\infty$.

Be specific please.

4. Complete: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots = \infty$. The name of this series is called the Harmonic series.

Write in the box an exact number or DNE or ∞ or $-\infty$.

Be specific please.

5. a. In sigma notation the series $f(x) = 1 - x + (-x)^2 + (-x)^3 + (-x)^4 + \dots = \sum_{n=0}^{\infty} (-x)^n$

b. The series $f(x)$ is a child of the geometric series $\sum_{k=0}^{\infty} ar^k$ where the value of $a = 1$ and $-x = r$.
The radius of convergence of $f(x)$ is $R = 1$.

c. At the left endpoint, $f(x)$ becomes the series $\sum_{k=0}^{\infty} (1)^k = 1 + 1 + 1 + 1 + \dots$ which will diverge.
 $f(-1) = \rightarrow$

What is true about the limit of partial sums S_n ? $\lim_{n \rightarrow \infty} S_n = \infty$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $f(x)$ at the left endpoint? vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point

d. At the right endpoint, $f(x)$ becomes the series $\sum_{k=0}^{\infty} (-1)^k = 1 - 1 + 1 - 1 + \dots$ which will diverge.
 $f(1) = \rightarrow$

What is true about the limit of partial sums S_n ? $\lim_{n \rightarrow \infty} S_n = \text{DNE}$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $f(x)$ at the right endpoint? vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point

e. Report the sum of the series $f(x)$ on its interval of convergence. $a = 1$, $r = -x$, $\frac{a}{1-r} = \frac{1}{1-(-x)} = \frac{1}{1+x}$

f. Use the Root Test or Ratio Test (your choice) to show that the radius of convergence in part b is what you have claimed.

ROOT: $\lim_{n \rightarrow \infty} |x^n|^{\frac{1}{n}} = \lim_{n \rightarrow \infty} |x| = |x| \cdot \lim_{n \rightarrow \infty} 1 = |x| < 1$ RATIO: $\lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{x^n} \right| = \lim_{n \rightarrow \infty} |x| = |x| < 1$

g. Integrate the above series term by term to create $g(x) = \int f(x) dx$. TIP: Be sure you are integrating.

$$g(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \dots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \cdot x^k}{k}$$

h. The radius of convergence of $g(x)$ is $R = 1$.

i. At the left endpoint, $g(x)$ becomes the series $\sum_{k=1}^{\infty} \frac{(-1)^{2k+1}}{k} = -1 + \frac{-1}{2} + \frac{-1}{3} + \frac{-1}{4} + \dots$ which will diverge.
 $g(-1) = \rightarrow$

What is true about the limit of partial sums S_n ? $\lim_{n \rightarrow \infty} S_n = -\infty$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $g(x)$ at the left endpoint? vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point

j. At the right endpoint, $g(x)$ becomes the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ which will converge.
 $g(1) = \rightarrow$ \rightarrow AST

What is true about the limit of partial sums S_n ? $\lim_{n \rightarrow \infty} S_n = \ln 2$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $g(x)$ at the right endpoint? vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point

k. Report the sum of the series $g(x)$ on its interval of convergence. TIP: Integrate the expression in 5e. $\int \frac{1}{1+x} dx = \ln(1+x)$

6. a. In sigma notation the series $u(x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \dots = \sum_{n=1}^{\infty} \boxed{-\frac{x^n}{n}}$

b. Use the series for $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ to find what function $u(x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \dots$ approximates.
 $u(x) = \boxed{\ln(1-x)}$. The radius of convergence is $R = \boxed{1}$. Simplified please.

Replace x with -x

c. Write the first four terms of the series in expanded form if $x = -1$. $\boxed{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}}$ - ...

The left endpoint $x = -1$ is in the interval of convergence. Explain your answer.

Reason: Alternating Harmonic Series (Question 3) A.S.T.

What is true about the graph of $u(x)$ at the left endpoint?

{hole, vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point}

d. Write the first four terms of the series in expanded form if $x = 1$. $\boxed{-1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4}}$ - ...

The right endpoint $x = 1$ is not in the interval of convergence. Explain your answer. Simplified please.

Reason: Negative of Harmonic Series (Question 4)

What is true about the graph of $u(x)$ at the right endpoint?

{hole, vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point}

7. Consider the series $v(x) = 32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \dots$

60.3, 60.5, 60.7, 60.9

a. Examine the pattern to report the next term: $32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \frac{\boxed{32x^{540}}}{9}$

b. The series $v(x)$ is a child series of the series $\tan^{-1}w = w - \frac{w^3}{3} + \frac{w^5}{5} - \frac{w^7}{7} + \dots = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{w^{2n+1}}{2n+1}$ which converges on $-1 \leq w \leq 1$. Use the series for $\tan^{-1}w$ to write what $v(x)$ converges to on its interval of convergence.

$v(x) = 32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \dots = \boxed{32 \tan^{-1}(x^{60})}$

c. Write the series $v(x)$ in sigma notation: $v(x) = \sum_{n=1}^{\infty} (-1)^n \cdot \boxed{32} \cdot \frac{\boxed{x^{60}}^{2n+1}}{2n+1} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{32x^{120n+60}}{2n+1}$

d. Differentiate the series $v(x)$ term by term to create the expanded series for $v'(x)$. TIP: Be sure you are differentiating.

$v'(x) = \boxed{1920x^{59}} + \boxed{-1920x^{179}} + \boxed{1920x^{299}} + \boxed{-1920x^{419}} + \dots$

e. The radius of convergence of $v'(x)$ is $R = \boxed{1}$. On its interval of convergence, $v'(x)$ converges to $v'(x) = \frac{\boxed{1920x^{59}}}{\boxed{1+x^{120}}}$. TIP: Differentiate the expression in part b.

f. Write the series $v'(x)$ in sigma notation: $v'(x) = \sum_{n=1}^{\infty} \boxed{1920x} \frac{\boxed{120x+59}}{\boxed{2n+1}}$

TIP: Differentiate the expression in part c.

Find $32 \cdot \frac{d}{dx} \left(\frac{(x^{60})^{2n+1}}{2n+1} \right) = 32 \cdot \frac{(60)(2n+1) x^{120n+60-1}}{(2n+1)} =$

$v' = 32 \tan^{-1}(x^{60})$
 $= 32 \cdot \frac{1}{1+(x^{60})^2} \cdot \frac{d}{dx} x^{60}$
 $= \frac{32 \cdot 60 x^{59}}{1+x^{120}}$

$$v'(x) = 1920x^{59} - 1920x^{179} + 1920x^{299} - 1920x^{419} + \dots$$

g. Write the first four terms of the series $v'(x)$ in expanded form if $x = -1$.

$$\boxed{-1920 + 1920 - 1920 + 1920 - \dots}$$

The left endpoint $x = -1$ is not in the interval of convergence. Explain your answer.

Simplified please.

Reason: diverges by oscillation

What is true about the graph of $v'(x)$ at the left endpoint?

hole vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point

h. Write the first four terms of the series $v'(x)$ in expanded form if $x = 1$.

$$\boxed{1920 - 1920 + 1920 - 1920 + \dots}$$

The right endpoint $x = 1$ is not in the interval of convergence. Explain your answer.

Simplified please.

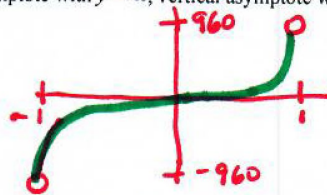
Reason: diverges by oscillation

What is true about the graph of $v'(x)$ at the right endpoint?

hole vertical asymptote with $y \rightarrow \infty$, vertical asymptote with $y \rightarrow -\infty$, defined point

d. Sketch a graph of the series $v'(x)$ on its the interval of convergence.

$$v'(x) = \frac{1920x^{59}}{1+x^{120}}$$



i. The term-by-term derivative of $f(x) = \sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \dots$ is the power series below.

a. Write the first four nonzero terms of the series for $f'(x)$.

Simplified please

$$f'(x) = \boxed{5 + 10x + 15x^2 + 20x^3} + \dots$$

b. The radius of convergence of $f'(x)$ is $R = \underline{1}$ This is a geometric series with $r = x$ and $Q = 5$ so it converges if $|x| < 1$

c. If x is equal to the **left endpoint** of the interval of convergence, the series for $f'(x)$ will diverge.

{converge, diverge}

$$x = -1 \Rightarrow 5 - 10 + 15 - 20 + \dots \text{ diverges by oscillation } s_n = 5, -5, 10, -10, 15, -15, \dots$$

e. If x is equal to the **right endpoint** of the interval of convergence, the series for $f'(x)$ will diverge.

{converge, diverge}

$$x = 1 \Rightarrow 5 + 10 + 15 + 20 + \dots = \infty \text{ (vertical asymptote)}$$

f. Write the series for $f'(x)$ in sigma notation.

$$f(x) = \sum 5x^n$$

$$f'(x) = \sum_{n=1}^{\infty} \boxed{5nx^{n-1}}$$

$$f'(x) = 5 \sum \frac{d}{dx} x^n = 5 \cdot \sum nx^{n-1}$$

g. When x is in the interval of convergence, we can write the series for $f'(x)$ as what rational function?

$$f'(x) = \frac{\boxed{5}}{\boxed{(1-x)^2}}$$

Differentiate the right hand side:

$$f(x) = 5 + 5x + 5x^2 + 5x^3 + \dots = \frac{5}{1-x} \text{ on } (-1, 1)$$

$$\begin{aligned} f'(x) &= 5 + 10x + 15x^2 + \dots = \frac{d}{dx} 5(1-x)^{-1} \\ &= -5(1-x)^{-2} \cdot \frac{d}{dx}(1-x) \\ &= \frac{-5}{(1-x)^2} \cdot (-1) \\ &= \frac{5}{(1-x)^2} \end{aligned}$$

h. Sketch a graph of $\sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \dots$ on its the interval of convergence.



hole at $x = -1$
v.a. at $x = 1$ approaching ∞

Chain rule