Practice Questions from HW 26

1. Consider the function $f(x) = \sum_{n=0}^{\infty} 30\left(\frac{x-40}{20}\right)^n = 30 + 30\left(\frac{x-40}{20}\right) + 30\left(\frac{x-40}{20}\right)^2 + 30\left(\frac{x-40}{20}\right)^3 + \ldots$

a. Evaluate. No work need be shown.
Write in the box an exact number or DNE or ∞ or $-\infty$.

$\frac{30}{1-.8} = \frac{30}{.2} = \boxed{150}$

$\sum 30(-1)^n = 30-30+30-30+\cdots$ diverges by oscillation

$f(72) = \boxed{\infty}$ $f(56) = \boxed{150}$ $f(20) = \boxed{DNE}$ $f(60) = \boxed{\infty}$

$\sum_{n=0}^{\infty} 30(1.6)^n = \infty$

$\sum_{n=0}^{\infty} 30(.8)^n$ $a = 30$ $r = .8$

$\sum 30(1)^n = 30 + 30 + \ldots$

b. For what values of x does $f(x)$ converge? Show work. $\boxed{20} < x < \boxed{60}$

$r > 1$ so diverges

$-1 < \frac{x-40}{20} < 1$

$-20 < x - 40 < 20$

$20 < x < 60$

c. Report the sum of the series on its interval of convergence.

$a = 30$, $r = \frac{x-40}{20}$ $\frac{a}{1-r} = \frac{30}{\left(1 - \left(\frac{x-40}{20}\right)\right)} \cdot \frac{20}{20} = \frac{600}{20-(x-40)} \Rightarrow \frac{600}{60-x}$

d. What is true about the graph of $f(x)$ at the left endpoint? At $x = 20$, since $f(20) = DNE$ we have a

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

e. What is true about the graph of $f(x)$ at the right endpoint? At $x = 60$, since $f(60) = \infty$, we have a

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

2. The series $c(x) = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot 2x^{5k}}{1024^k}$ is a child of the geometric series $\sum_{k=0}^{\infty} ar^k$ where the value of $a = \boxed{2}$ and $\boxed{\frac{-x^5}{1024}} = r$

which converges for $\boxed{-4} < x < \boxed{4}$. You can solve the inequality graphically or with a table.

a. At the left endpoint, $c(x)$ becomes the series $\sum_{k=0}^{\infty} \boxed{2 \cdot 1^k} = \boxed{2} + \boxed{2} + \boxed{2} + \boxed{2} + \ldots$ which will ______.
{converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n = \boxed{\infty}$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $c(x)$ at the left endpoint? ______

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

b. At the right endpoint, $c(x)$ becomes the series $\sum_{k=0}^{\infty} \boxed{2 \cdot (-1)^k} = \boxed{2} + \boxed{-2} + \boxed{2} + \boxed{-2} + \ldots$ which will ______.
{converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n = \boxed{DNE}$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $c(x)$ at the right endpoint? ______

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

c. Report the sum of the series on its interval of convergence.

$a = 2$
$r = \frac{-x^5}{1024}$

$\frac{a}{1-r} = \frac{2}{\left(1 + \frac{x^5}{1024}\right)} \cdot \frac{1024}{1024} = \boxed{\frac{2048}{1024 + x^5}}$

3. Complete: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots = \boxed{\ln 2}$. The name of this series is called the Alternating Harmonic series.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

4. Complete: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots = \boxed{\infty}$. The name of this series is called the Harmonic series.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

5. **a.** In sigma notation the series $f(x) = 1 - x + (-x)^2 + (-x)^3 + (-x)^4 + \dots = \sum_{n=0}^{\infty} \boxed{(-x)^n}$

b. The series $f(x)$ is a child of the geometric series $\sum_{k=0}^{\infty} ar^k$ where the value of $a = \boxed{1}$ and $\boxed{-x} = r$

The radius of convergence of $f(x)$ is $R = $ 1.

c. At the left endpoint, $f(x)$ becomes the series $f(-1) = \sum_{k=0}^{\infty} \boxed{(1)^n} = \boxed{1} + \boxed{1} + \boxed{1} + \boxed{1} + \dots$ which will ________ {converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n = \boxed{\infty}$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $f(x)$ at the left endpoint? ________

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

d. At the right endpoint, $f(x)$ becomes the series $f(1) = \sum_{k=0}^{\infty} \boxed{(-1)^n} = \boxed{1} + \boxed{-1} + \boxed{1} + \boxed{-1} + \dots$ which will ________ {converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n = \boxed{\text{DNE}}$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $f(x)$ at the right endpoint? ________

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

e. Report the sum of the series $f(x)$ on its interval of convergence. $a = 1$, $r = -x$ $\quad \frac{a}{1-r} = \frac{1}{1-(-x)} = \frac{1}{1+x}$

f. Use the Root Test or Ratio Test (your choice) to show that the radius of convergence in part **b** is what you have claimed.

ROOT: $\lim_{n\to\infty} |x^n|^{\frac{1}{n}} = \lim_{n\to\infty} |x| = |x| \cdot \lim_{n\to\infty} 1 = |x| < 1$

RATIO: $\lim_{n\to\infty} \left| \frac{x^{n+1}}{x^n} \right| = \lim_{n\to\infty} |x| = |x| < 1$

g. Integrate the above series term by term to create $g(x) = \int f(x)dx$. TIP: Be sure you are integrating.

$g(x) = \boxed{x} + \boxed{\frac{-x^2}{2}} + \boxed{\frac{x^3}{3}} + \boxed{\frac{-x^4}{4}} + \boxed{\frac{x^5}{5}} + \dots = \sum_{k=1}^{\infty} \boxed{(-1)^{k+1} \cdot \frac{x^k}{k}}$

h. The radius of convergence of $g(x)$ is $R = $ 1.

i. At the left endpoint, $g(x)$ becomes the series $g(-1) = \sum_{k=1}^{\infty} \boxed{\frac{(-1)^{2k+1}}{k}} = \boxed{-1} + \boxed{\frac{-1}{2}} + \boxed{\frac{-1}{3}} + \boxed{\frac{-1}{4}} + \dots$ which will ________ {converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n = \boxed{-\infty}$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $g(x)$ at the left endpoint? ________

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

j. At the right endpoint, $g(x)$ becomes the series $g(1) = \sum_{k=1}^{\infty} \boxed{\frac{(-1)^{k+1}}{k}} = \boxed{1} + \boxed{\frac{-1}{2}} + \boxed{\frac{1}{3}} + \boxed{\frac{-1}{4}} + \dots$ which will ________ AST {converge, diverge}

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n = \boxed{\ln 2}$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $g(x)$ at the right endpoint? ________

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

k. Report the sum of the series $g(x)$ on its interval of convergence. TIP: Integrate the expression in **5e**. $\int \frac{1}{1+x} dx = \ln(1+x)$

6. a. In sigma notation the series $u(x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \dots = \sum_{\boxed{n=1}}^{\infty} \boxed{-\frac{x^n}{n}}$

b. Use the series for $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ to find what function $u(x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \dots$ approximates.

$u(x) = \boxed{\ln(1-x)}$. The radius of convergence is $R = \boxed{1}$

Replace x with $-x$

Simplified please.

c. Write the first four terms of the series in expanded form if $x = -1$. $\boxed{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}}$ $- \dots$

The left endpoint $x = -1$ ______ in the interval of convergence. Explain your answer.
{is, is not}

Reason: Alternating Harmonic Series (Question 3) A.S.T.

What is true about the graph of $u(x)$ at the left endpoint? ______
{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

d. Write the first four terms of the series in expanded form if $x = 1$. $\boxed{-1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4}}$ $- \dots$

Simplified please.

The right endpoint $x = 1$ ______ in the interval of convergence. Explain your answer.
{is, is not}

Reason: Negative of Harmonic Series (Question 4)

What is true about the graph of $u(x)$ at the right endpoint? ______
{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

7. Consider the series $v(x) = 32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \dots$

$60 \cdot 3$ $\quad 60 \cdot 5$ $\quad 60 \cdot 7$ $\quad 60 \cdot 9$

a. Examine the pattern to report the next term: $32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \boxed{\frac{32x^{540}}{9}}$

b. The series $v(x)$ is a child series of the series $\tan^{-1} w = w - \frac{w^3}{3} + \frac{w^5}{5} - \frac{w^7}{7} + \dots = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{w^{2n+1}}{2n+1}$ which converges on $-1 \le w \le 1$. Use the series for $\tan^{-1} w$ to write what $v(x)$ converges to on its interval of convergence.

$v(x) = 32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \dots = \boxed{32\tan^{-1}(x^{60})}$

c. Write the series $v(x)$ in sigma notation: $v(x) = \sum_{n=1}^{\infty} (-1)^n \cdot \boxed{32} \cdot \frac{\left(\boxed{x^{60}}\right)^{2n+1}}{2n+1} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{32x^{120n+60}}{2n+1}$

d. Differentiate the series $v(x)$ term by term to create the expanded series for $v'(x)$. TIP: Be sure you are differentiating.

$v'(x) = \boxed{1920x^{59}} + \boxed{-1920x^{179}} + \boxed{1920x^{299}} + \boxed{-1920x^{419}} + \dots$

e. The radius of convergence of $v'(x)$ is $R =$ $\underline{1}$. On its interval of convergence, $v'(x)$ converges to $v'(x) = \boxed{\frac{1920x^{59}}{1+x^{120}}}$
TIP: Differentiate the expression in part **b.**

f. Write the series $v'(x)$ in sigma notation: $v'(x) = \sum_{n=1}^{\infty} \boxed{1920x^{120x+59}}$

TIP: Differentiate the expression in part **c.**

Find $32 \cdot \frac{d}{dx}\left(\frac{(x^{60})^{2n+1}}{2n+1}\right) = 32 \cdot \frac{(60)\cancel{(2n+1)}\, x^{120n+60-1}}{\cancel{(2n+1)}} =$

$v' = 32\tan^{-1}(x^{60})$

$= 32 \cdot \frac{1}{1+(x^{60})^2} \cdot \frac{d}{dx} x^{60}$

$= \frac{32 \cdot 60x^{59}}{1+x^{120}}$

$v'(x) = 1920x^{59} - 1920x^{179} + 1920x^{299} - 1920x^{419}$

g. Write the first four terms of the series $v'(x)$ in expanded form if $x = -1$. $\boxed{-1920 + 1920 - 1920 + 1920 - \cdots}$ Simplified please.

The left endpoint $x = -1$ _________ in the interval of convergence. Explain your answer.
{is, is not}

Reason: diverges by oscillation

What is true about the graph of $v'(x)$ at the left endpoint? ________________
{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

h. Write the first four terms of the series $v'(x)$ in expanded form if $x = 1$. $\boxed{1920 - 1920 + 1920 - 1920}$ $+ \cdots$ Simplified please.

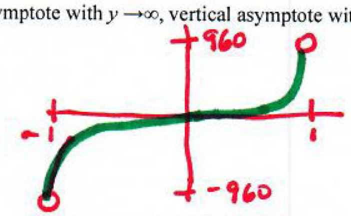
The right endpoint $x = 1$ _________ in the interval of convergence. Explain your answer.
{is, is not}

Reason: diverges by oscillation

What is true about the graph of $v'(x)$ at the right endpoint? ________________
{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

d. Sketch a graph of the series $v'(x)$ on its the interval of convergence.

$$v'(x) = \frac{1920x^{59}}{1 + x^{120}}$$



3. The term-by-term derivative of $f(x) = \sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ is the power series below.

a. Write the first four nonzero terms of the series for $f'(x)$. Simplified please

$f'(x) = \boxed{5 + 10x + 15x^2 + 20x^3} + \ldots$

b. The radius of convergence of $f'(x)$ is $R =$ 1 This is a geometric series with $r = x$ and $a = 5$ so it converges if $|x| < 1$

c. If x is equal to the **left endpoint** of the interval of convergence, the series for $f'(x)$ will __________.
{converge, diverge}

$x = -1 \Rightarrow 5 - 10 + 15 - 20 + \cdots$ diverges by oscillation $S_n = 5, -5, 10, -10, 15, -15, \ldots$

e. If x is equal to the **right endpoint** of the interval of convergence, the series for $f'(x)$ will __________.
{converge, diverge}

$x = 1 \Rightarrow 5 + 10 + 15 + 20 + \cdots = \infty$ (vertical asymptote)

f. Write the series for $f'(x)$ in sigma notation.

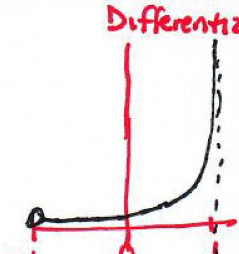
$f'(x) = \sum_{n=\boxed{1}}^{\infty} \left(\boxed{5nx^{n-1}} \right)$

$f(x) = \sum 5x^n$

$f'(x) = 5 \sum \frac{d}{dx} x^n = 5 \cdot \sum n x^{n-1}$

g. When x is in the interval of convergence, we can write the series for $f'(x)$ as what rational function?

$f'(x) = \dfrac{\boxed{5}}{\boxed{(1-x)^2}}$



Differentiate the right hand side:

$f(x) = 5 + 5x + 5x^2 + 5x^3 + \cdots = \frac{5}{1-x}$ on $(-1, 1)$

$f'(x) = 5 + 10x + 15x^2 + \cdots = \frac{d}{dx} 5(1-x)^{-1}$

$= -5(1-x)^{-2} \cdot \frac{d}{dx}(1-x)$

$= \frac{-5}{(1-x)^2} \cdot (-1)$

$= \frac{5}{(1-x)^2}$

Chain rule

h. Sketch a graph of $\sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ on its the interval of convergence.

hole at $x = -1$
V.A at $x = 1$ approaching ∞