Practice Questions from HW 24

1. Consider the function $f(x) = \sum_{n=0}^{\infty} 30\left(\frac{x-40}{20}\right)^n = 30 + 30\left(\frac{x-40}{20}\right) + 30\left(\frac{x-40}{20}\right)^2 + 30\left(\frac{x-40}{20}\right)^3 + \ldots$

   a. Evaluate. No work need be shown.
   Write in the box an exact number or DNE or ∞ or $-\infty$.

   $f(72) =$ [∞] $f(56) =$ [150] $f(20) =$ [DNE] $f(60) =$ [∞]

   $\frac{30}{1-.8} = \frac{30}{.2} = 150$ $\quad \sum 30(-1)^n = 30-30+30-30+\ldots$ diverges by oscillation $\quad \sum 30(1)^n = 30+30+\ldots.$ $\quad \sum_{n=0}^{\infty} 30(1.6)^n = \infty$ $\quad \sum_{n=0}^{\infty} 30(.8)^n$ $a=30\ r=.8$

   b. For what values of x does $f(x)$ converge? Show work. [20] $< x <$ [60) $\quad -1 < \frac{x-40}{20} < 1$ $\quad -20 < x-40 < 20$ $\quad 20 < x < 60$

   $r > 1$ so diverges

   c. Report the sum of the series on its interval of convergence. $a=30,\ r=\frac{x-40}{20}$ $\quad \frac{a}{1-r} = \frac{30}{1-\left(\frac{x-40}{20}\right)} \cdot \frac{20}{20} = \frac{600}{20-(x-40)} = \frac{600}{60-x}$

   d. What is true about the graph of $f(x)$ at the left endpoint? _____
   At $x=20$, since $f(20)=$ DNE we have a {hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

   e. What is true about the graph of $f(x)$ at the right endpoint? _____
   At $x=60$, since $f(60)=\infty$, we have a {hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

2. The series $c(x) = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot 2x^{5k}}{1024^k}$ is a child of the geometric series $\sum_{k=0}^{\infty} ar^k$ where the value of $a =$ [2] and [$\frac{-x^5}{1024}$] $= r$

   which converges for [-4] $< x <$ [4]. You can solve the inequality graphically or with a table.

   a. At the left endpoint, $c(x)$ becomes the series $\sum_{k=0}^{\infty}$ [$2 \cdot 1^k$] $=$ [2] $+$ [2] $+$ [2] $+$ [2] $+ \ldots$ which will _____.
   {converge, diverge}

   What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n =$ [∞]. Write in the box an exact number or DNE or ∞ or $-\infty$.

   What is true about the graph of $c(x)$ at the left endpoint? _____
   {hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

   b. At the right endpoint, $c(x)$ becomes the series $\sum_{k=0}^{\infty}$ [$2 \cdot (-1)^k$] $=$ [2] $+$ [-2] $+$ [2] $+$ [-2] $+ \ldots$ which will _____.
   {converge, diverge}

   What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n =$ [DNE]. Write in the box an exact number or DNE or ∞ or $-\infty$.

   What is true about the graph of $c(x)$ at the right endpoint? _____
   {hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

   c. Report the sum of the series on its interval of convergence. $a=2$ $\quad r = \frac{-x^5}{1024}$ $\quad \frac{a}{1-r} = \frac{2}{\left(1+\frac{x^5}{1024}\right)} \cdot \frac{1024}{1024} = \frac{2048}{1024+x^5}$

3. Complete: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots =$ [$\ln 2$]. The name of this series is called the Alternating Harmonic series.
   Write in the box an **exact** number or DNE or ∞ or $-\infty$.
   Be specific please.

4. Complete: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \ldots =$ [∞]. The name of this series is called the Harmonic series.
   Write in the box an **exact** number or DNE or ∞ or $-\infty$.
   Be specific please.

5. **a.** In sigma notation the series $f(x) = 1 - x + (-x)^2 + (-x)^3 + (-x)^4 + \ldots = \sum_{n=0}^{\infty} (-x)^n$

b. The series $f(x)$ is a child of the geometric series $\sum_{k=0}^{\infty} ar^k$ where the value of $a =$ 1 and $r =$ $-x$

The radius of convergence of $f(x)$ is $R =$ 1.

c. At the left endpoint, $f(x)$ becomes the series $\sum_{k=0}^{\infty} (1)^n = 1 + 1 + 1 + 1 + \ldots$ which will {converge, diverge}.

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n =$ ∞. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $f(x)$ at the left endpoint? {hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

d. At the right endpoint, $f(x)$ becomes the series $\sum_{k=0}^{\infty} (-1)^n = 1 + (-1) + 1 + (-1) + \ldots$ which will {converge, diverge}.

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n =$ DNE. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $f(x)$ at the right endpoint? {hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

e. Report the sum of the series $f(x)$ on its interval of convergence. $a = 1$, $r = -x$, $\frac{a}{1-r} = \frac{1}{1-(-x)} = \frac{1}{1+x}$

f. Use the Root Test or Ratio Test (your choice) to show that the radius of convergence in part **b** is what you have claimed.

ROOT: $\lim_{n \to \infty} |x^n|^{\frac{1}{n}} = \lim_{n \to \infty} |x| = |x| \cdot \lim_{n \to \infty} 1 = |x| < 1$

RATIO: $\lim_{n \to \infty} \left| \frac{x^{n+1}}{x^n} \right| = \lim_{n \to \infty} |x| = |x| < 1$

g. Integrate the above series term by term to create $g(x) = \int f(x)\,dx$. TIP: Be sure you are integrating.

$g(x) = x + \frac{-x^2}{2} + \frac{x^3}{3} + \frac{-x^4}{4} + \frac{x^5}{5} + \ldots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \cdot x^k}{k}$

h. The radius of convergence of $g(x)$ is $R =$ 1.

i. At the left endpoint, $g(x)$ becomes the series $\sum_{k=1}^{\infty} \frac{(-1)^{2k+1}}{k} = -1 + \frac{-1}{2} + \frac{-1}{3} + \frac{-1}{4} + \ldots$ which will {converge, diverge}.

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n =$ $-\infty$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $g(x)$ at the left endpoint? {hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

j. At the right endpoint, $g(x)$ becomes the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 + \frac{-1}{2} + \frac{1}{3} + \frac{-1}{4} + \ldots$ which will {converge, diverge} by AST.

What is true about the limit of partial sums S_n? $\lim_{n \to \infty} S_n =$ $\ln 2$. Write in the box an exact number or DNE or ∞ or $-\infty$.

What is true about the graph of $g(x)$ at the right endpoint? {hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

k. Report the sum of the series $g(x)$ on its interval of convergence. TIP: Integrate the expression in **5e**. $\int \frac{1}{1+x}\,dx = \ln(1+x)$

6. a. In sigma notation the series $u(x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots = \sum_{n=1}^{\infty} \boxed{-\frac{x^n}{n}}$

b. Use the series for $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ to find what function $u(x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots$ approximates.

$u(x) =$ $\boxed{\ln(1-x)}$. The radius of convergence is $R =$ $\boxed{1}$

Replace x with $-x$

c. Write the first four terms of the series in expanded form if $x = -1$. $\boxed{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}} - \ldots$ Simplified please.

The left endpoint $x = -1$ ~~is~~ / is not in the interval of convergence. Explain your answer.

Reason: Alternating Harmonic Series (Question 3) A.S.T.

What is true about the graph of $u(x)$ at the left endpoint? {hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, ~~defined point~~}

d. Write the first four terms of the series in expanded form if $x = 1$. $\boxed{-1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4}} - \ldots$ Simplified please.

The right endpoint $x = 1$ ~~is~~ / is not in the interval of convergence. Explain your answer.

Reason: Negative of Harmonic Series (Question 4)

What is true about the graph of $u(x)$ at the right endpoint? {hole, vertical asymptote with $y \to \infty$, ~~vertical asymptote with $y \to -\infty$~~, defined point}

7. Consider the series $v(x) = 32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \ldots$ (60.3, 60.5, 60.7, 60.9)

a. Examine the pattern to report the next term: $32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \boxed{\frac{32x^{540}}{9}}$

b. The series $v(x)$ is a child series of the series $\tan^{-1} w = w - \frac{w^3}{3} + \frac{w^5}{5} - \frac{w^7}{7} + \ldots = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{w^{2n+1}}{2n+1}$

which converges on $-1 \le w \le 1$. Use the series for $\tan^{-1} w$ to write what $v(x)$ converges to on its interval of convergence.

$v(x) = 32x^{60} - \frac{32x^{180}}{3} + \frac{32x^{300}}{5} - \frac{32x^{420}}{7} + \ldots =$ $\boxed{32\tan^{-1}(x^{60})}$

c. Write the series $v(x)$ in sigma notation: $v(x) = \sum_{n=1}^{\infty} (-1)^n \cdot \boxed{32} \cdot \frac{\boxed{x^{60}}^{2n+1}}{2n+1} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{32x^{120n+60}}{2n+1}$

d. Differentiate the series $v(x)$ term by term to create the expanded series for $v'(x)$. TIP: Be sure you are differentiating.

$v'(x) =$ $\boxed{1920x^{59}} + \boxed{-1920x^{179}} + \boxed{1920x^{299}} + \boxed{-1920x^{419}} + \ldots$

e. The radius of convergence of $v'(x)$ is $R =$ $\underline{1}$. On its interval of convergence, $v'(x)$ converges to $v'(x) =$ $\boxed{\frac{1920x^{59}}{1+x^{120}}}$

TIP: Differentiate the expression in part b.

f. Write the series $v'(x)$ in sigma notation: $v'(x) = \sum_{n=1}^{\infty} \boxed{1920x^{120n+59}}$

TIP: Differentiate the expression in part c.

Find $32 \cdot \frac{d}{dx}\left(\frac{(x^{60})^{2n+1}}{2n+1}\right) = 32 \cdot \frac{(60)(2n+1)\,x^{120n+60-1}}{(2n+1)} =$

$v' = 32\tan^{-1}(x^{60})$

$= 32 \cdot \frac{1}{1+(x^{60})^2} \cdot \frac{d}{dx} x^{60}$

$= \frac{32 \cdot 60x^{59}}{1+x^{120}}$

$v'(x) = 1920x^{59} - 1920x^{179} + 1920x^{299} - 1920x^{419}$

g. Write the first four terms of the series $v'(x)$ in expanded form if $x = -1$. $-1920 + 1920 - 1920 + 1920 - \cdots$ Simplified please.

The left endpoint $x = -1$ {is, is not} in the interval of convergence. Explain your answer.

Reason: diverges by oscillation

What is true about the graph of $v'(x)$ at the left endpoint?

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

h. Write the first four terms of the series $v'(x)$ in expanded form if $x = 1$. $1920 - 1920 + 1920 - 1920 + \cdots$ Simplified please.

The right endpoint $x = 1$ {is, is not} in the interval of convergence. Explain your answer.

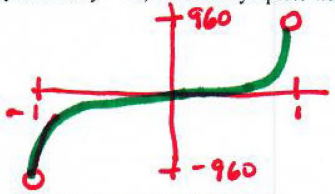
Reason: diverges by oscillation

What is true about the graph of $v'(x)$ at the right endpoint?

{hole, vertical asymptote with $y \to \infty$, vertical asymptote with $y \to -\infty$, defined point}

d. Sketch a graph of the series $v'(x)$ on its the interval of convergence.

$v'(x) = \dfrac{1920x^{59}}{1 + x^{120}}$



3. The term-by-term derivative of $f(x) = \sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ is the power series below.

a. Write the first four nonzero terms of the series for $f'(x)$. Simplified please

$f'(x) = 5 + 10x + 15x^2 + 20x^3 + \ldots$

b. The radius of convergence of $f'(x)$ is $R = 1$ This is a geometric series with $r = x$ and $a = 5$ so it converges if $|x| < 1$

c. If x is equal to the **left endpoint** of the interval of convergence, the series for $f'(x)$ will {converge, diverge}.

$x = -1 \Rightarrow 5 - 10 + 15 - 20 + \cdots$ diverges by oscillation $s_n = 5, -5, 10, -10, 15, -15, \ldots$

e. If x is equal to the **right endpoint** of the interval of convergence, the series for $f'(x)$ will {converge, diverge}.

$x = 1 \Rightarrow 5 + 10 + 15 + 20 + \cdots = \infty$ (vertical asymptote)

f. Write the series for $f'(x)$ in sigma notation.

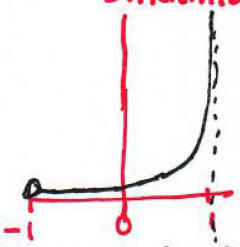
$f'(x) = \sum_{n=1}^{\infty} 5nx^{n-1}$

$f(x) = \sum 5x^n$

$f'(x) = 5 \sum \frac{d}{dx} x^n = 5 \cdot \sum n x^{n-1}$

g. When x is in the interval of convergence, we can write the series for $f'(x)$ as what rational function?

$f'(x) = \dfrac{5}{(1-x)^2}$



Differentiate the right hand side:

$f(x) = 5 + 5x + 5x^2 + 5x^3 + \cdots = \frac{5}{1-x}$ on $(-1, 1)$

$f'(x) = 5 + 10x + 15x^2 + \cdots = \frac{d}{dx} 5(1-x)^{-1}$

$= -5(1-x)^{-2} \cdot \frac{d}{dx}(1-x)$

$= \frac{-5}{(1-x)^2} \cdot (-1)$

$= \frac{5}{(1-x)^2}$

Chain rule

h. Sketch a graph of $\sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ on its the interval of convergence.

hole at $x = -1$

v.a at $x = 1$ approaching ∞

9. Suppose a series $\sum_{n=1}^{\infty}$ (some expression involving w and n) converges to a function $f(w)$ on an interval. Complete.

a. Suppose when w is replaced by a number the sum of the series **converges**.
Then at that value of w the graph of $f(w)$ has a _____.
{open hole, closed hole, vertical asymptote approaching $-\infty$, vertical asymptote approaching ∞}

b. Suppose when w is replaced by a number the sum of the series **diverges by oscillation**.
Then at that value of w the graph of $f(w)$ has a _____.
{open hole, closed hole, vertical asymptote approaching $-\infty$, vertical asymptote approaching ∞}

c Suppose when w is replaced by a number the sum of the series **diverges off to $-\infty$**.
Then at that value of w the graph of $f(w)$ has a _____.
{open hole, closed hole, vertical asymptote approaching $-\infty$, vertical asymptote approaching ∞}

d. Suppose when w is replaced by a number the sum of the series **diverges off to ∞**.
Then at that value of w the graph of $f(w)$ has a _____.
{open hole, closed hole, vertical asymptote approaching $-\infty$, vertical asymptote approaching ∞}

e. $\sum_{n=1}^{\infty}(-1)^n = -1+1-1+1-1+\ldots$ **DNE** Write in the box an **exact** number or DNE or ∞ or $-\infty$.

See the lesson or recording on ***Fruit From a Fertile Father*** on Tuesday, November 4 why this is DNE.

f. $\sum_{n=1}^{\infty}(-1)^{2n+2} = \sum_{n=1}^{\infty}\left((-1)^2\right)^{n+1} = \infty$ Write in the last box an **exact** number or DNE or ∞ or $-\infty$.

This series is $\sum_{n=1}^{\infty} 1 = 1+1+1+1+\ldots = \infty$

g. $-11\sum_{n=1}^{\infty}(1)^n = -11-11-11-\ldots = -\infty$ Write in the box an **exact** number or DNE or ∞ or $-\infty$.

h. $11\sum_{n=1}^{\infty}(1)^n = 11+11+11+\ldots = \infty$ Write in the box an **exact** number or DNE or ∞ or $-\infty$.

10. For Question **6** you found the series for $u(x) = -x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}-\frac{x^5}{5}-\ldots = \ln(1-x)$ on $-1 \le x < 1$.

a. Write the series $u(x)$ in sigma notation. $u(x) = -\sum_{n=1}^{\infty}\frac{x^n}{n}$

b. Create the new series $h(x) = u'(x)$ by differentiation $u(x)$ term by term.

$h(x) = u'(x) =$ **-1** $-$ **x** $-$ **x^2** $-$ **x^3** $-\ldots$

c. Write the series $h(x)$ in sigma notation. You can differentiate your expression in **10a**. Check it matches **10b**.

$-\sum_{n=1}^{\infty}x^{n-1}$ or $-\sum_{k=0}^{\infty}x^k$ where $k = n-1$.

d. **i.** Find the (non-series) representation by differentiating $\ln(1-x)$. $h(x) = \frac{-1}{1-x}$.
ii. The radius of convergence of $h(x) = u'(x)$ is $R =$ **1**.
iii. If x is equal to the **left endpoint** of the interval of convergence, the series for $h(x) = u'(x)$ will _____ and will
{converge, diverge}
become $h($**-1**$) = u'(x) =$ **-1** $+$ **1** $+$ **-1** $+$ **1** $+\ldots =$ **DNE** Write in the last box an **exact** number or DNE or ∞ or $-\infty$.

At this endpoint the graph of $h(x) = u'(x)$ will have a _____.
{open hole, closed hole, vertical asymptote approaching $-\infty$, vertical asymptote approaching ∞}

iv. If x is equal to the **right endpoint** of the interval of convergence, the series for $h(x) = u'(x)$ will _____ and will
{converge, diverge}

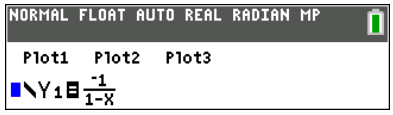
become $h($ **1** $) = u'(x) =$ **−1** $+$ **−1** $+$ **−1** $+$ **−1** $+\ldots =$ **−∞** Write in the last box an **exact** number or DNE or ∞ or −∞.

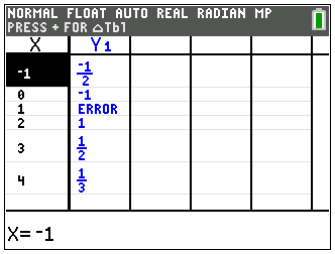
At this endpoint the graph of $h(x) = u'(x)$ will have a _____.
{open hole, closed hole, vertical asymptote approaching −∞, vertical asymptote approaching ∞}

v. If the graph has an open hole, report its coordinates: (**−1** , **−1/2**).

Substitute −1 in the expression $\frac{-1}{1-x}$. The y-coordinate of the hole is $\frac{-1}{1-(-1)} = \frac{-1}{1+1} = -\frac{1}{2}$.

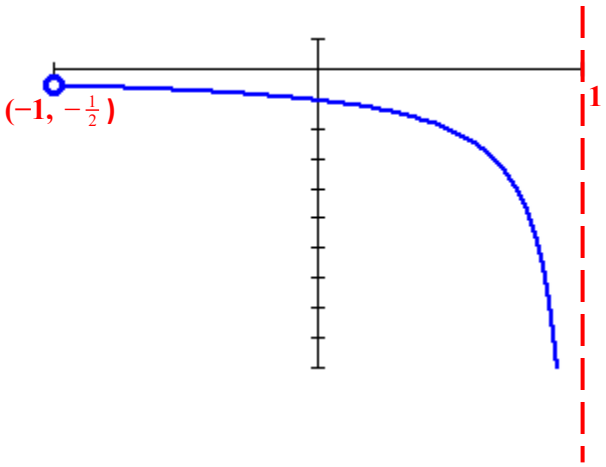
You could use the table feature of a grapher.





e. Report the interval of convergence. **(−1, 1) or $|x| < 1$ or $-1 < x < 1$.**

f. Sketch a graph on its interval of convergence. A grapher may help.



g. Show how you could create the terms of the series $h(x)$ as a child of the series for $\sum_{n=0}^{\infty} x^n$ ***without*** using differentiation, but instead by using either substitution (of some expression involving x) and algebra (multiplication of each term by an expression involving x and real numbers).

$h(x) = -\sum_{k=0}^{\infty} x^k = \frac{-1}{1-x}$ is a vertical reflection of $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.

We multiply the terms by −1.