MA 16600 KEY Practice Questions over 8.3 and Appendix C

1. State the double-angle identity used to integrate $\sin^2 x$. $\sin^2 x = \boxed{\frac{1}{2}(1 - \cos 2x)}$

2. State the double-angle identity used to integrate $\cos^2 x$. $\cos^2 x = \boxed{\frac{1}{2}(1 + \cos 2x)}$

3. Find the indefinite integral. Show work. $\int \sin^3 x\, dx$

$$\int \sin^3 x\, dx = \int \sin^2 x \sin x dx$$

$$= \int \left(1 - \cos^2 x\right) \sin x dx$$

$$= \int \sin x dx \quad - \quad (-1) \int \cos^2 x \cdot (-\sin x)\, dx$$

Let $u = \cos x$
$du = -\sin x dx$

$$= \int \sin x dx \quad + \quad \int u^2 du$$

$$= -\cos x \quad + \quad \frac{u^3}{3} \quad + \quad C$$

$$= -\cos x \quad + \quad \frac{\cos^3 x}{3} \quad + \quad C$$

$$= \frac{\cos^3 x}{3} \quad - \quad \cos x \quad + \quad C$$

4. Find the indefinite integrals. Show work.

a. $\int \tan^9 x \sec^2 x\, dx =$ $\frac{1}{10}\tan^{10} x$ $+ C$

$u = \tan x$
$du = \sec^2 x dx$ $\Rightarrow \int \tan^9 x \sec^2 x dx = \int u^9 du = \frac{1}{10} u^{10} + C$
$= \boxed{\frac{1}{10}\tan^{10} x + C}$

b. $\int \cos^2 \theta\, d\theta =$ $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$ $+ C$

Use the Pythagorean Double Identity $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$\int \cos^2\theta d\theta = \frac{1}{2}\int (1 + \cos 2\theta) d\theta = \frac{1}{2}\int d\theta + \frac{1}{2}\cdot\frac{1}{2}\int \underbrace{\cos 2\theta}_{\cos u} \cdot \underbrace{2 d\theta}_{du}$

$= \boxed{\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C}$

c. $\int \sin^3 x \cos^6 x\, dx = \underline{\frac{1}{9}\cos^9 x - \frac{1}{7}\cos^7 x} + C$

If you let $u = \sin x$, then we need one $\cos x$ for du, which leaves $\int u^3 \cos^5 x\, dx$ and Pythagoras can't help us unless it is even.

So let $u = \cos x$, $du = -\sin x\, dx$ $\Rightarrow \int \cos^6 x \cdot \sin^2 x \cdot \underbrace{\sin x\, dx\,(-1)}_{du}(-1)$

$= \int u^6 \cdot \sin^2 x \cdot du \cdot (-1)$

Pythagoras to the rescue! $\sin^2 x = 1 - \cos^2 x = 1 - u^2$

$= \int u^6 \cdot (-1)(1-\cos^2 x)\, du$

$= \int u^6 (-1)(1-u^2)\, du = \int u^6(u^2-1)\, du$

$= \int (u^8 - u^6)\, du$

$= \frac{1}{9}u^9 - \frac{1}{7}u^7 + C = \boxed{\frac{1}{9}\cos^9 x - \frac{1}{7}\cos^7 x + C}$

5. Consider the integral $\int \frac{\sin\theta}{\cos^2\theta}\, d\theta$.

a. Select which of these is the antiderivative for the integral $\int \frac{\sin\theta}{\cos^2\theta}\, d\theta$.

A. $\sin\theta + C$	B. $\cos\theta + C$	C. $\tan\theta + C$	D. $\csc\theta + C$	E. $\sec\theta + C$	F. $\cot\theta + C$
G. $-\sin\theta + C$	H. $-\cos\theta + C$	I. $-\tan\theta + C$	J. $-\csc\theta + C$	K. $-\sec\theta + C$	L. $-\cot\theta + C$
M. All of these.	N. None of these.				

b. Explain your reasoning for your selection.

Method 1: $\int \frac{\sin\theta}{\cos^2\theta}\, d\theta = \int \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta}\, d\theta = \int \sec\theta \tan\theta\, d\theta = \boxed{\sec\theta + C}$ from formula sheet

Method 2: Let $u = \cos\theta$, $du = -\sin\theta\, d\theta$ $\quad \int \frac{\sin\theta}{\cos^2\theta}\, d\theta = -\int \frac{1}{\cos^2\theta}(-\sin\theta)\, d\theta = -\int u^{-2}\, du = \frac{1}{u} + C$

$= \frac{1}{\cos\theta} + C$

$= \boxed{\sec\theta + C}$

6. Consider $\int \sec^{14} x \tan^{17} x\, dx$

a. Suppose we let $u = \tan x$. Then $du = \underline{\sec^2 x}\, dx$

Then we can write $\int \sec^{14} x \tan^{17} x\, dx = \int \boxed{u^{17}(1+u^2)^6}\, du$.

Your answer is a binomial in terms of u raised to a power multiplied by u raised to a power.
Do not multiply it out. Do not find the antiderivative. Just leave it as a polynomial.

$\int \sec^{14} x \tan^{17} x\, dx = \int u^{17} \sec^{12} x \cdot \underbrace{\sec^2 x\, dx}_{du}$

$= \int u^{17} (\sec^2 x)^6\, du$

$= \int u^{17} (1 + \tan^2 x)^6\, du = \int u^{17}(1+u^2)^6\, du$

use $1 + \tan^2 x = \sec^2 x$

b. Suppose we let $w = \sec x$. Then $dw = \underline{\sec w \tan w}\, dx$

Then we can write $\int \sec^{14} x \tan^{17} x\, dx = \int \boxed{w^{13}(w^2-1)^8}\, dw$.

Your answer is a binomial in terms of w raised to a power multiplied by w raised to a power.
Do not multiply it out. Do not find the antiderivative. Just leave it as a polynomial.

$\int \sec^{14} x \tan^{17} x\, dx = \int \sec^{13} x \tan^{16} x \underbrace{\sec x \tan x\, dx}_{du}$

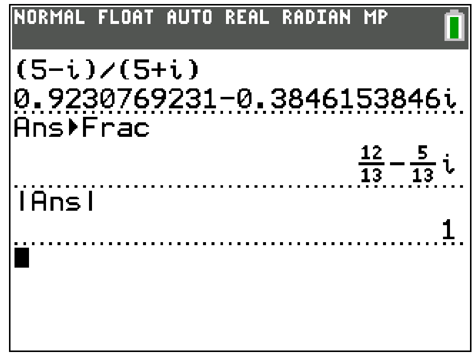
$= \int w^{13} (\tan^2 x)^8\, du = \int w^{13}(w^2-1)^8\, dw$

use $\sec^2 x - 1 = \tan^2 x$

7. a. Use your calculator to compute $z = \dfrac{5-i}{5+i}$ and report in the form $z = x + yi$, where x and y are **exact** real numbers.

Then find the absolute value of z.

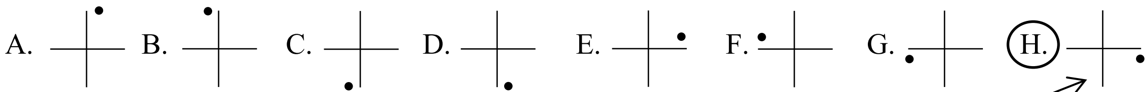
$$z = \frac{5-i}{5+i} = \boxed{\frac{12}{13}} + \boxed{\frac{-5}{13}}\, i$$



b. $|z| = \left|\dfrac{5-i}{5+i}\right| = \boxed{1}$ (Use your calculator's absolute value command.)

Without a calculator you create $\sqrt{\left(\frac{12}{13}\right)^2 + \left(\frac{5}{13}\right)^2}$

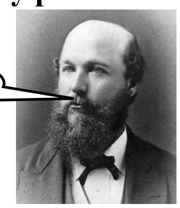
c. Determine the location of z on the complex plane. Assume a square grid. (Select one)



The point is in the fourth quadrant since it has a positive real part and negative imaginary part. Since the real part 12/13 is larger than |−5/13| , z looks most like Choice H.

d. Write the complex number z in polar form $r\text{cis }\theta$, where r is an exact real number. Approximate θ in degrees to 1 decimal place.

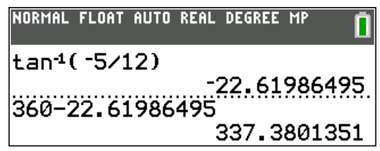
$r\text{cis }\theta = r(\cos\theta + i\sin\theta)$


Washington Irving Stringham

$$z = \frac{5-i}{5+i} = \boxed{1}\ \text{cis}\ \boxed{-22.6^\circ}$$

$$\theta = \tan^{-1}\frac{-5/13}{12/13} = \tan^{-1}\frac{-5}{12}$$

We expect a small angle.



Also $z = \text{cis } 337.4^\circ$ is correct.

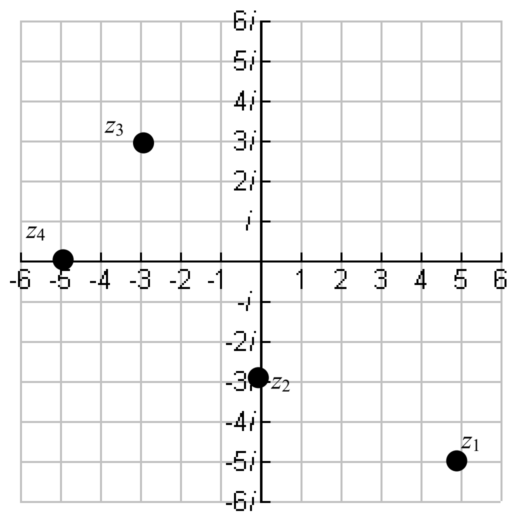
8. a. Plot and **label** the complex numbers on the grid shown.

$z_1 = 5 - 5i$

$z_2 = -3i$

$z_3 = 3\sqrt{2}\text{ cis }\dfrac{3\pi}{4}$

$z_4 = 5\text{cis }7\pi$



b. Write z_1 and z_2 in polar form $r\text{cis }\theta$, where r and θ are exact real numbers (and θ is in radians). Hint: Part (a) may help. (There are many correct answers for θ; however, report exact **radians** please.)

For θ please choose from 0, π/6, π/4, π/3, π/2, 2π/3, 3π/4, 5π/6, π, 7π/6, 5π/4, 4π/3, 3π/2, 5π/3, 7π/4, or 11π/6

$z_1 = 5 - 5i$

$r = \underline{5\sqrt{2}}$

$\theta = \underline{7\pi/4}$

Polar form $r\text{cis}\theta$ of $z_1 = 5 - 5i = \boxed{5\sqrt{2}}\ \text{cis}\ \boxed{7\pi/4}$

$z_2 = -3i$

$r = \underline{3}$

$\theta = \underline{3\pi/2}$

Polar form $r\text{cis}\theta$ of $z_2 = -3i = \boxed{3}\ \text{cis}\ \boxed{3\pi/2}$

c. Write z_3 and z_4 in rectangular form $x + yi$, where x and y are **exact** real numbers.

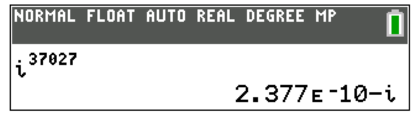
Stretch each leg of the 1-1-$\sqrt{2}$ isosceles right triangle by 3. Or multiply $3\sqrt{2}$ by $-\frac{\sqrt{2}}{2}$ and by $\frac{\sqrt{2}}{2}$ or use a calculator.

$$3\sqrt{2}\cdot\frac{-\sqrt{2}}{2} = -3 \qquad 3\sqrt{2}\cdot\frac{\sqrt{2}}{2} = 3$$

$$z_3 = 3\sqrt{2}\text{ cis }\frac{3\pi}{4} = \boxed{-3} + \boxed{3}\cdot i$$

$$z_4 = 5\text{cis }7\pi = \boxed{-5} + \boxed{0}\cdot i$$

9. Consider the complex number i^{37027}.

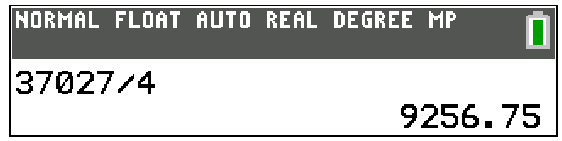


a. A student uses a calculator to try to write the number in rectangular form $x + yi$, where x and y are real numbers. See the screen shown.
What should the exact answer really be? Report the exact answer in rectangular form $x + yi$:

$$i^{37027} = \boxed{0} + \boxed{-1} \cdot i$$

The value 2.377E−10 is the calculator's confusing attempt at reporting the number 0.

$$2.377 \cdot 10^{-10} = 0.0000000002377$$



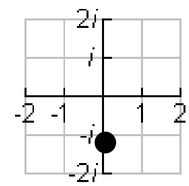
Manually, you can pull out a multiple of 4 from 37027 since $i^4 = 1$.

So $9256 \cdot 4 = 37024$ is a close multiple of 4 near 37027.

$$i^{37027} = i^{37024}\, i^3 = (i^4)^{9256} \cdot i^3 = (1)^{9256} \cdot i^3 = (1)i^3 = i^2 i = -i \; = 0 - i$$

b. Report the location of i^{37027} in the complex plane. Plot the point.

A. the positive real axis B. the positive imaginary axis C. the negative real axis
D. the negative imaginary axis E. none of these



c. Use part **b** to report i^{37027} in polar form in polar form $r\text{cis}\ \theta$, where r and θ are exact real numbers (and θ is in radians). Hint: Part (b) may help. (There are many correct answers for θ; however for θ please choose from 0, π/6, π/4, π/3, π/2, 2π/3, 3π/4, 5π/6, π, 7π/6, 5π/4, 4π/3, 3π/2, 5π/3, 7π/4, or 11π/6)

Polar form $r\text{cis}\,\theta$ of $i^{37027} = \boxed{1}\ \text{cis}\ \boxed{3\pi/2}$