Practice Questions Over Sections 8.2 -8.4

1. Integrate by parts. Show work. $\int x \sin 5x\, dx = \boxed{-\frac{1}{5}x\cos 5x + \frac{1}{26}\sin 5x} + C$

take derivative

$u =$ x $\quad dv =$ $\sin 5x$ dx

$du =$ 1 $dx \quad v =$ $-\frac{1}{5}\cos 5x$

integrate. Remember $\int \sin 5x\, dx$
$= \frac{1}{5}\int \sin 5x\, 5dx$
$= \frac{1}{5}(-\cos 5x) + C$
You can check by differentiation

$\int \underbrace{x}_{u} \underbrace{\sin 5x\, dx}_{dv} = uv - \int v\, du$

$= -\frac{1}{5}x\cos 5x - -\frac{1}{5}\int \cos 5x\, dx$

$= -\frac{1}{5}x\cos 5x + \frac{1}{5}\cdot\frac{1}{5}\int \cos 5x \cdot 5dx =$

check:
$-\frac{1}{5}\frac{d}{dx}x\cos 5x + \frac{1}{25}\frac{d}{dx}\sin 5x$
$= -\frac{1}{5}(x\cdot(-5\sin 5x) + \cos 5x) + \frac{1}{25}(5\cos 5x)$
$= x\sin 5x - \frac{1}{5}\cos 5x + \frac{1}{5}\cos 5x = x\sin 5x$

2. Integrate by parts. Show work. $\int xe^{-x}\, dx = \boxed{-xe^{-x} - e^{-x}} + C$

take derivative

$u =$ x $\quad dv =$ e^{-x} dx

$du =$ 1 $dx \quad v =$ $-e^{-x}$

integrate

$\int u\,dv = uv - \int v\,du$
$= -xe^{-x} - -\int e^{-x}dx$
$= -xe^{-x} - \int e^{-x}(-dx)$
$= -xe^{-x} - e^{-x} + C$

just did this

check: $-\frac{d}{dx}xe^{-x} - \frac{d}{dx}e^{-x}$
$= -(x\cdot(-e^{-x}) + e^{-x}) - (-e^{-x})$
$= xe^{-x} - e^{-x} + e^{-x} = xe^{-x}$

3. Integrate by parts. Show work. $\int x \ln x\, dx = \boxed{\frac{1}{2}x^2\ln x - \frac{1}{4}x^2} + C$

take derivative

$u =$ $\ln x$ $\quad dv =$ x dx

$du =$ $\frac{1}{x}$ $dx \quad v =$ $\frac{x^2}{2}$

integrate

$\int u\,dv = uv - \int v\,du$
$= \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx$
$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\, dx$
$= \frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{1}{2}x^2 + C$
$= \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C$

just did this

check: $\frac{1}{2}\frac{d}{dx}(x^2\cdot\frac{1}{x} + 2x\cdot\ln x) - \frac{1}{4}\frac{d}{dx}x^2$
$= \frac{1}{2}\cdot(x + 2x\ln x) - \frac{1}{4}\cdot 2x$
$= \frac{1}{2}x + x\ln x - \frac{1}{2}x$
$= x\ln x$

4. Find the indefinite integrals. Show work.

a. $\int \tan^9 x \sec^2 x\, dx =$ $\frac{1}{10}\tan^{10}x$ $+ C$

$u = \tan x$, $du = \sec^2 x dx$ $\Rightarrow \int \tan^9 x \sec^2 x dx = \int u^9 du = \frac{1}{10}u^{10} + C = \boxed{\frac{1}{10}\tan^{10}x + C}$

b. $\int \frac{\sec\theta}{\tan^2\theta} d\theta =$ $-\csc\theta$ $+ C$

Method 1: $\int \frac{\sec\theta}{\tan^2\theta} d\theta = \int \sec\theta \frac{1}{\tan^2\theta} d\theta = \int \frac{1}{\cos\theta} \cdot \frac{1}{\frac{\sin\theta}{\cos\theta}} \cdot \frac{1}{\tan\theta} d\theta = \int \frac{1}{\sin\theta} \cdot \frac{1}{\tan\theta} d\theta$

$= \int \csc\theta \cot\theta d\theta = \boxed{-\csc\theta} + C$ from formula sheet

Method 2: $\int \frac{\sec\theta}{\tan^2\theta} d\theta = \int \frac{1}{\cos\theta} \cdot \cot^2\theta d\theta = \int \frac{1}{\cos\theta} \frac{\cos^2\theta}{\sin^2\theta} d\theta = \int \frac{\cos\theta}{\sin^2\theta} d\theta$ This is like #5 below

$u = \sin\theta$, $du = \cos\theta d\theta$

$= \int \frac{1}{u^2} \cdot \cos\theta d\theta = \int \frac{1}{u^2} du = -\frac{1}{u} + C = \frac{-1}{\sin\theta} + C = \boxed{-\csc\theta + C}$

c. $\int \cos^2\theta\, d\theta =$ $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$ $+ C$

Use the Pythagorean Double Identity $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$\int \cos^2\theta d\theta = \frac{1}{2}\int (1 + \cos 2\theta) d\theta = \frac{1}{2}\int d\theta + \frac{1}{2} \cdot \frac{1}{2} \int \underbrace{\cos 2\theta}_{\cos u} \cdot \underbrace{2 d\theta}_{du}$

$= \boxed{\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C}$

d. $\int \sin^3 x \cos^6 x\, dx =$ $\frac{1}{9}\cos^9 x - \frac{1}{7}\cos^7 x$ $+ C$

If you let $u = \sin x$, then we need one $\cos x$ for du, which leaves $\int u^3 \cos^5 x dx$ and Pythagoras can't help us unless it is even.

So let $u = \cos x$, $du = -\sin x dx$ $\Rightarrow \int \cos^6 x \cdot \sin^2 x \cdot \sin x dx (-1)(-1)$

$= \int u^6 \cdot \sin^2 x \cdot du \cdot (-1)$

Pythagoras to the rescue! $\sin^2 x = 1 - \cos^2 x = 1 - u^2$

$= \int u^6 \cdot (-1)(1 - \cos^2 x) du$

$= \int u^6 (-1)(1 - u^2) du = \int u^6 (u^2 - 1) du$

$= \int (u^8 - u^6) du$

$= \frac{1}{9}u^9 - \frac{1}{7}u^7 + C = \boxed{\frac{1}{9}\cos^9 x - \frac{1}{7}\cos^7 x + C}$

5. Consider the integral $\int \frac{\sin\theta}{\cos^2\theta} d\theta$.

a. Select which of these is the antiderivative for the integral $\int \frac{\sin\theta}{\cos^2\theta} d\theta$.

A. $\sin\theta + C$	B. $\cos\theta + C$	C. $\tan\theta + C$	D. $\csc\theta + C$	E. $\sec\theta + C$	F. $\cot\theta + C$
G. $-\sin\theta + C$	H. $-\cos\theta + C$	I. $-\tan\theta + C$	J. $-\csc\theta + C$	K. $-\sec\theta + C$	L. $-\cot\theta + C$
M. All of these.	N. None of these.				

b. Explain your reasoning for your selection.

Method 1: $\int \frac{\sin\theta}{\cos^2\theta} d\theta = \int \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} d\theta = \int \sec\theta \tan\theta d\theta = \boxed{\sec\theta + C}$ from formula sheet

Method 2: Let $u = \cos\theta$, $du = -\sin\theta d\theta$ $\quad \int \frac{\sin\theta}{\cos^2\theta} d\theta = -\int \frac{1}{\cos^2\theta} (-\sin\theta) d\theta = -\int u^{-2} du = \frac{1}{u} + C$

$= \frac{1}{\cos\theta} + C$

$= \boxed{\sec\theta + C}$

6. Consider $\int \sec^{14} x \tan^{17} x \, dx$

(2) **a.** Suppose we let $u = \tan x$. Then $du =$ $\sec^2 x$ dx

Then we can write $\int \sec^{14} x \tan^{17} x \, dx = \int \boxed{u^{17}(1+u^2)^6} \, du.$

Your answer is a binomial in terms of u raised to a power multiplied by u raised to a power.
Do not multiply it out. Do not find the antiderivative. Just leave it as a polynomial.

$$\int \sec^{14}x \tan^{17}x\,dx = \int u^{17}\sec^{12}x \cdot \underbrace{\sec^2 x\,dx}_{du}$$
$$= \int u^{17}(\sec^2 x)^6 \, du$$
$$= \int u^{17}(1+\tan^2 x)^6 du = \int u^{17}(1+u^2)^6 du$$

use $1+\tan^2 x = \sec^2 x$

(2) **b.** Suppose we let $w = \sec x$. Then $dw =$ $\sec w \tan w$ dx

Then we can write $\int \sec^{14} x \tan^{17} x \, dx = \int \boxed{w^{13}(w^2-1)^8} \, dw.$

Your answer is a binomial in terms of w raised to a power multiplied by w raised to a power.
Do not multiply it out. Do not find the antiderivative. Just leave it as a polynomial.

$$\int \sec^{14}x \tan^{17}x\,dx = \int \sec^{13}x \tan^{16}x \underbrace{\sec x \tan x\,dx}$$
$$= \int w^{13}(\tan^2 x)^8 \; du = \int w^{13}(w^2-1)^8 dw$$

use $\sec^2 x - 1 = \tan^2 x$

The quiz will contain a bonus question on trig substitution. Here are some for practice.

7. Integrate $\int \frac{25}{x^2\sqrt{x^2-25}} dx$, $x > 5$ using trig substitution.

$$\sqrt{x^2-25} = \sqrt{(5\sec\theta)^2-25} = \sqrt{25}\sqrt{\sec^2\theta - 1} = 5\sqrt{\tan^2\theta} = 5\tan\theta$$

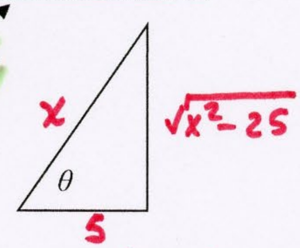
a. Complete: $x = 5\sec\theta$ $dx =$ $5\sec\theta\tan\theta$ $d\theta$, $\sqrt{x^2-25} =$ $5\tan\theta$.

b. Write entirely in terms of θ. Simplify your answer in the boxes as much as possible. Show work.

$$\int \frac{25}{x^2\sqrt{x^2-25}} dx = \int \left(\boxed{\cos\theta} \right) d\theta = \boxed{\sin\theta} + C$$

Involves θ

$$\int \frac{25}{x^2\sqrt{x^2-25}} dx = \int \frac{25}{(5\sec\theta)^2 \cdot (5\tan\theta)} \cdot 5\sec\theta\tan\theta\,d\theta$$
$$= \int \frac{\cancel{25}\cdot\cancel{5}\sec\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{25}\sec^2\theta \cdot \cancel{5\tan\theta}} = \int \frac{1}{\sec\theta} \cdot d\theta$$
$$= \int \cos\theta\,d\theta$$
$$= \sin\theta + c$$



$5\sec\theta = x$

$\sec\theta = \frac{x}{5}$

$\cos\theta = \frac{5}{x} = \frac{ADJ}{HYP}$

$\sin\theta = \frac{OPP}{HYP} = \frac{\sqrt{x^2-25}}{x}$

c. Write entirely in terms of x. Label the right triangle to help you. Show work.

$$\int \frac{25}{x^2\sqrt{x^2-25}} dx = \boxed{\frac{\sqrt{x^2-25}}{x}} + C$$

Involves x

8. Integrate $\int \frac{x}{\sqrt{16-x^2}}\,dx$ using trig substitution.

$\sqrt{16-x^2} = \sqrt{16-(4\sin\theta)^2} = \sqrt{16}\sqrt{1-\sin^2\theta} = 4\sqrt{\cos^2\theta} = 4\cos\theta$

a. Complete: $x = 4\sin\theta$ $\quad dx =$ $4\cos\theta$ $d\theta$, $\sqrt{16-x^2} =$ $4\cos\theta$.

b. Write entirely in terms of θ. Simplify your answer in the boxes as much as possible.

$$\int \frac{x}{\sqrt{16-x^2}}\,dx = \int \left(\boxed{4\sin\theta}\right)d\theta = \boxed{-4\cos\theta} + C$$

Involves θ

$$\int \frac{x}{\sqrt{16-x^2}}\,dx = \int \frac{(4\sin\theta)}{(4\cos\theta)}\cdot(4\cos\theta\,d\theta)$$

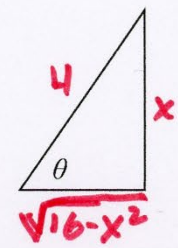
$$= \int \frac{\cancel{4}\sin\theta\cdot 4\cancel{\cos\theta}}{\cancel{4}\cancel{\cos\theta}}\,d\theta = \int 4\sin\theta\,d\theta = -4\cos\theta + C$$

c. Write entirely in terms of x. Label the right triangle and use it to help you. Show work.

$$\int \frac{x}{\sqrt{16-x^2}}\,dx = \boxed{-\sqrt{16-x^2}} + C$$

Involves x

$-4\cos\theta = -4\cdot\frac{\sqrt{16-x^2}}{4}$



$4\sin\theta = x$

$\sin\theta = \frac{x}{4} = \frac{OPP}{HYP}$

$\cos\theta = \frac{ADJ}{HYP} = \frac{\sqrt{16-x^2}}{4}$

9. Integrate $\int \frac{x^2\,dx}{(x^2+36)^{3/2}}$ using trig substitution.

$\sqrt{x^2+36} = \sqrt{(6\tan\theta)^2+36} = \sqrt{36(\tan^2\theta+1)} = \sqrt{36}\sqrt{\sec^2\theta} = 6\sec\theta$

a. Complete: $x = 6\tan\theta$ $\quad dx =$ $6\sec^2\theta$ $d\theta$, $\sqrt{x^2+36} =$ $6\sec\theta$ $\quad (x^2+36)^{3/2} =$ $(6\sec\theta)^3$

$= (x^2+36)^{1/2}$ $\qquad = ((x^2+36)^{1/2})^3$

b. Write entirely in terms of θ. Simplify your answer in the boxes as much as possible.

$$\int \frac{x^2\,dx}{(x^2+36)^{3/2}} = \int \left(\boxed{\sec\theta - \tan\theta}\right)d\theta = \boxed{\ln|\sec\theta+\tan\theta| - \sin\theta} + C$$

Involves θ

$$\int \frac{x^2}{(x^2+36)^{3/2}}\,dx = \int \frac{(6\tan\theta)^2}{(6\sec\theta)^3}\cdot(6\sec^2\theta\,d\theta)$$

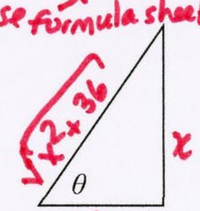
$$= \int \frac{\cancel{6^2}\tan^2\theta}{\cancel{6^3}\sec^3\theta}\cdot\cancel{6}\cdot\sec^2\theta\,d\theta = \int \frac{\tan^2\theta}{\sec\theta}\,d\theta = \int \frac{(\sec^2\theta-1)}{\sec\theta}\,d\theta = \int\left(\frac{\sec^2\theta}{\sec\theta} - \frac{1}{\sec\theta}\right)d\theta = \int(\sec\theta - \cos\theta)\,d\theta$$

$= \int \sec\theta\,d\theta - \int\cos\theta\,d\theta$

Use $\tan^2\theta = \sec^2\theta - 1$ $\qquad$ Divide $\qquad$ Use formula sheet

c. Write entirely in terms of x. Label the right triangle and use it to help you. Show work.

$$\int \frac{x^2\,dx}{(x^2+36)^{3/2}} = \boxed{\ln\left|\frac{\sqrt{x^2+36}}{6} + \frac{x}{6}\right| - \frac{x}{\sqrt{x^2+36}}} + C$$



Involves x

$\cos\theta = \frac{ADJ}{HYP} = \frac{6}{\sqrt{x^2+36}}$ // $\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{x^2+36}}{6}$ // $\tan\theta = \frac{x}{6}$ // $\sin\theta = \frac{OPP}{HYP} = \frac{x}{\sqrt{x^2+36}}$