Practice Questions from Section 10.5 and Section 10.6

Insert numbers or expressions with the correct variables in the boxes. Circle the correct choice in the word bank.

1. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{7/4}}$.

a. We can use the Comparison Test with $b_n = \boxed{\frac{1}{n^{7/4}}}$ to show the series $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{7/4}}$ __________
{converges, diverges}

b. Which is true for your choice of b_n?

Circle one: **i.** $\frac{\sin^2 n}{n^{7/4}} \leq b_n$ **ii.** $b_n \leq \frac{\sin^2 n}{n^{7/4}}$

c. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will __________ because **p-series with p = 7/4**
{converge, diverge}
(Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as p-series, harmonic series, geometric series, etc.)

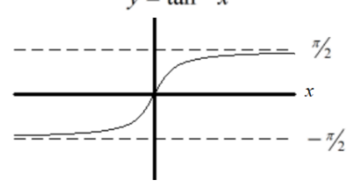
2. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{\tan^{-1} n}{3^n}$.

a. We can use the Comparison Test with $b_n = \boxed{\frac{\left(\frac{\pi}{2}\right)}{3^n}}$ to show the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{3^n}$ __________
{converges, diverges}

b. Which is true for your choice of b_n?

Circle one: **i.** $\frac{\tan^{-1} n}{3^n} \leq b_n$ **ii.** $b_n \leq \frac{\tan^{-1} n}{3^n}$

$y = \tan^{-1} x$



c. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will __________ because **geometric series with r = 1/3**
{converge, diverge}
(Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as p-series, harmonic series, geometric series, etc.)

3. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{n}{n^2 - \cos^2 n}$.

a. We can use the Comparison Test with $b_n = \boxed{\frac{n}{n^2} = \frac{1}{n}}$ to show the series $\sum_{n=1}^{\infty} \frac{n}{n^2 - \cos^2 n}$ __________
{converges, diverges}

b. Which is true for your choice of b_n?

Circle one: **i.** $\frac{n}{n^2 - \cos^2 n} \leq b_n$ **ii.** $b_n \leq \frac{n}{n^2 - \cos^2 n}$

$n^2 > n^2 - \cos^2 n$ so $\frac{n}{n^2} < \frac{n}{n^2 - \cos^2 n}$

$\frac{1}{n} < \frac{n}{n^2 - \cos^2 n}$

c. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will __________ because **harmonic series**
{converge, diverge}
(Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as p-series, harmonic series, geometric series, etc.)

4. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{6n^2+n+7}{n^5+2n}$.

i. We can use the Limit Comparison Test with $b_n = \boxed{\frac{n^2}{n^5} = \frac{1}{n^3}}$ to show the series $\sum_{n=1}^{\infty} \frac{6n^2+n+7}{n^5+2n}$ ______ {converges, diverges}

ii. Complete, assuming b_n is what you wrote in the box in part **i**.

$\sum_{n=1}^{\infty} b_n$ will ______ {converge, diverge} because ***p*-series with *p* = 3**

(Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as p-series, harmonic series, geometric series, etc.)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{6}$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{(6n^2+n+7)}{(n^5+2n)} \cdot \frac{n^3}{} = 6$$

5. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{5n^5+8n^2}{\sqrt{n^{12}+8n^2}}$.

i. We can use the Limit Comparison Test with $b_n = \boxed{\frac{n^5}{n^6} = \frac{1}{n}}$ to show the series $\sum_{n=1}^{\infty} \frac{5n^5+8n^2}{\sqrt{n^{12}+8n^2}}$ ______ {converges, diverges}

ii. Complete, assuming b_n is what you wrote in the box in part **i**.

$\sum_{n=1}^{\infty} b_n$ will ______ {converge, diverge} because **harmonic series**

(Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as p-series, harmonic series, geometric series, etc.)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{5}$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{5n^5+8n^2}{\sqrt{n^{12}+8n^2}} \cdot \frac{n}{} = 5$$

6. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=4}^{\infty} \frac{1}{7\sqrt{n^3-4n+16}}$.

i. We can use the Limit Comparison Test with $b_n = \boxed{\frac{1}{n^{3/2}}}$ to show the series $\sum_{n=4}^{\infty} \frac{1}{7\sqrt{n^3-4n+16}}$ ______ {converges, diverges}

ii. Complete, assuming b_n is what you wrote in the box in part **i**.

$\sum_{n=1}^{\infty} b_n$ will ______ {converge, diverge} because ***p*-series with *p* = 1.5**

(Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as p-series, harmonic series, geometric series, etc.)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{\frac{1}{7}}$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{7\sqrt{n^3-4n+16}} \cdot \frac{n^{3/2}}{} = \frac{1}{7}$$

7. Suppose $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = \frac{2}{1} - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \frac{6}{5} - \frac{7}{6} + \ldots$

a. What is a_n? $a_n = \boxed{\frac{n+1}{n}}$

b. The series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ will ______.
{converge absolutely, converge conditionally, diverge }

c. Give a reason for your claim in part **b**. Since $\lim_{n\to\infty} \frac{n+1}{n} = 1$ then $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n+1}{n}$ diverges by the AST.

Alternatively, the series $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n+1}{n}$ has $\lim_{n\to\infty} (-1)^{n+1} \cdot \frac{n+1}{n} \neq 0$. (It diverges by oscillation to 1 and −1.)

Thus $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n+1}{n}$ diverges by the nth Term Test for Divergence.

8. Each alternating series below converges by the Alternating Series Test (AST).
Determine if the convergence is conditional or absolute.

a. $\sum_{n=1}^{\infty} \frac{(-1)^n 7n}{4n^3-3}$ will converge ______ because
{ absolutely, conditionally }

the series $\sum_{n=1}^{\infty} \frac{7n}{4n^3-3}$ will ______ by the ______ with $b_n = \boxed{\frac{n}{n^3} = \frac{1}{n^2}}$
{converge, diverge} {Comparison Test, Limit Comparison Test}

Provide the details of your claim below. Use $a_n = \frac{7n}{4n^3-3}$ and $b_n = \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{7n}{4n^3-3} \cdot \frac{n^2}{} = \frac{7}{4}$$

Thus $\sum_{n=1}^{\infty} \frac{7n}{4n^3-3}$ converges by the LCT. Since $\sum_{n=1}^{\infty} \frac{7n}{4n^3-3}$ converges, $\sum_{n=1}^{\infty} \frac{(-1)^n 7n}{4n^3-3}$ converges absolutely.

b. $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = \frac{1}{15} - \frac{1}{20} + \frac{1}{25} - \frac{1}{30} + \frac{1}{35} - \frac{1}{40} + \ldots$ will converge ______ because
{ absolutely, conditionally }

the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \boxed{\frac{1}{5n+10}}$ will ______ by the ______ with $b_n = \boxed{\frac{1}{n}}$.
{converge, diverge} {Comparison Test, Limit Comparison Test}

Provide the details of your claim below. Use $a_n = \frac{1}{5n+10}$ and $b_n = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{5n+10} \cdot \frac{n}{} = \frac{1}{5}$$

Thus $\sum_{n=1}^{\infty} \frac{1}{5n+10}$ diverges by the LCT, $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n+1}$ converges conditionally.

(Since $\lim_{n\to\infty} \frac{1}{5n+10} = 0$ and $a_n = \frac{1}{5n+10}$ decreases, $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n+1}$ converges by the AST.)

9. Report the two conditions for an alternating series $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} (-1)^{n+1} a_n$ to converge, where a_n is positive for all n.

i. **a_n is non-increasing**

ii. **$a_n \to 0$ as $n \to \infty$ (or $\lim_{n\to\infty} a_n = 0$)**

10. Give an example of a divergent alternating series with the property that its nth term approaches 0.
There are many correct answers. Hint: think of your answer to Question **9**.
You may write it in long form (expanded form) or use sigma notation, but use correct notation.

$$1 - \frac{1}{10} + \frac{1}{2} - \frac{1}{100} + \frac{1}{3} - \frac{1}{1000} + \frac{1}{4} - \frac{1}{10^4} + \frac{1}{5} - \frac{1}{10^5} + \frac{1}{6} - \frac{1}{10^6} + \ldots$$