Key to Practice Questions from Section 10.5

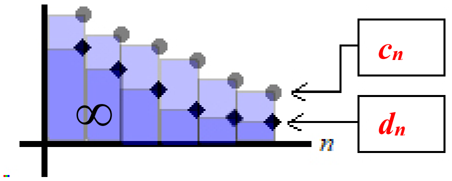
1. Complete the boxes and blanks. You can just circling the correct choices in the word bank.

a. i. To use the Direct Comparison Test to show the series $\sum_{n=1}^{\infty} c_n$ diverges,

we find a ______________ series $\sum_{n=1}^{\infty} d_n$ and must show that, for some large enough n, we have c_n ______ d_n.

{convergent, [✓ divergent]} { ≤, [✓ ≥]}

ii. Below is a plot of (n, c_n) and (n, d_n). Insert in the box which is c_n and which is d_n.

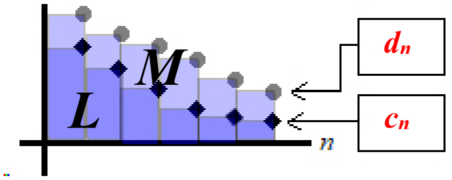


b. i. To use the Direct Comparison Test to show the series $\sum_{n=1}^{\infty} c_n$ converges,

we find a ______________ series $\sum_{n=1}^{\infty} d_n$ and must show that, for some large enough n, we have c_n ______ d_n.

{[✓ convergent], divergent} {[✓ ≤], ≥ }

ii. Below is a plot of (n, c_n) and (n, d_n). Insert in the box which is c_n and which is d_n.

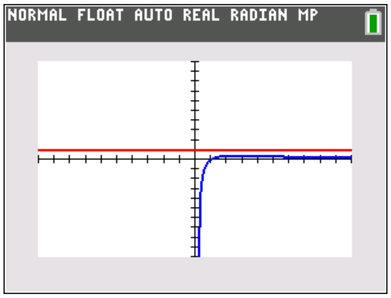


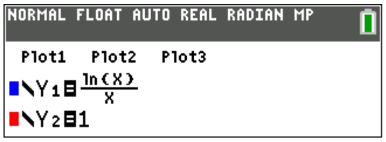
c. i. Suppose Finn decides to use the Direct Comparison Test to investigate the series $\sum_{n=1}^{\infty} \frac{\ln n}{n}$.

Complete the boxes. Finn chooses to compare $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ to the series $\sum_{n=1}^{\infty} 1$.

He tells you, "Since ln $n < n$, then $\frac{\ln n}{n} < \frac{n}{n} = 1$" You respond: [✓ A.]

B.





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 Plot1  Plot2  Plot3
■\Y1■ln(X)/X
■\Y2■1
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Finn goes on to say "We know that $\sum_{n=1}^{\infty} 1 = \boxed{\infty}$ because of the ________________________________."

(Give a reason for your answer such as harmonic series, p-series with $p > 1$, p-series with $p < 1$, geometric series, [✓ nth Term Test for Divergence], etc.)

He concludes "Therefore by the Direct Comparison Test we know $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ diverges."

You respond: A.

[✓ B.]

Finn's conclusion is fishy.
A series with a sum smaller than one that diverges could have either a finite sum (converge) or infinite sum (diverge).

ii. What alternative Convergence Test(s) could Finn use to show $\sum_{n=1}^{\infty}\frac{\ln n}{n}$ diverges? ______________
Select all possible answers.
(p-series with $p > 1$, p-series with $p < 1$, geometric series, [✓ Integral Test], nth Term Test)

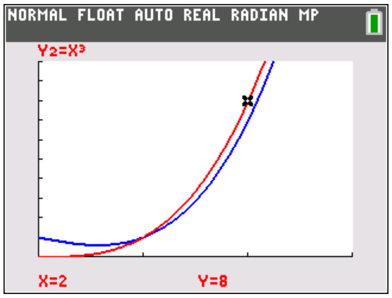
$\sum_{n=1}^{\infty}\frac{\ln n}{n}$ **diverges by the Integral Test since** $\int_{1}^{\infty} ln\, x\cdot\frac{1}{x}dx = \lim_{b\to\infty}\frac{\ln^2 b}{2} = \infty$.

d. **i.** Suppose Gil decides to use the Direct Comparison Test to investigate the series $\sum_{n=1}^{\infty}\left(\frac{n}{n^3-n+1}\right)$.

Complete the boxes. Gil compares $\sum_{n=1}^{\infty}\left(\frac{n}{n^3-n+1}\right)$ to the p-series $\sum_{n=1}^{\infty}\frac{1}{n^2} = \boxed{\frac{\pi^2}{6}}$

Gil tells you, "Since $n^3 - n + 1 \le n^3$ for $n \ge 1$, then $\frac{1}{n^2} = \frac{n}{n^3} \le \frac{n}{n^3-n+1}$." You say: [✓ A.]

B.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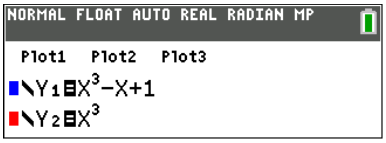
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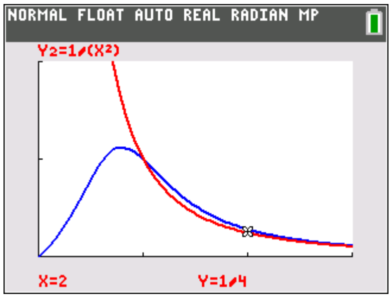
Plot1 Plot2 Plot3
$Y_1 = X^3-X+1$
$Y_2 = X^3$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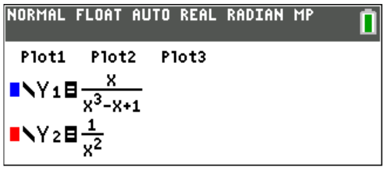
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Plot1 Plot2 Plot3
$Y_1 = \frac{X}{X^3-X+1}$
$Y_2 = \frac{1}{X^2}$

Gil concludes "Therefore by the Direct Comparison Test we know $\sum_{n=1}^{\infty}\left(\frac{n}{n^3-n+1}\right)$ converges."

You respond: A.

[✓ B.]

Gil's conclusion is fishy.
A series with a sum larger than one that converges could have either a finite sum (converge) or infinite sum (diverge).

ii. What alternative Convergence Test(s) could Gil use to show $\sum_{n=1}^{\infty}\left(\frac{n}{n^3-n+1}\right)$ converges? ______________

Select all possible answers.
(Geometric series, [✓ Limit Comparison Test], Integral Test, nth Term Test for Divergence)

$\sum_{n=1}^{\infty}\left(\frac{n}{n^3-n+1}\right)$ **converges by the Limit Comparison Test with** $a_n = \frac{n}{n^3-n+1}$ **and** $b_n = \frac{1}{n^2}$.

$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{a_n}{1}\cdot\frac{1}{b_n} = \lim_{n\to\infty}\frac{n}{n^3-n+1}\cdot\frac{n^2}{1} = 1$**. Since** $\sum_{n=1}^{\infty}\frac{1}{n^2}$ **converges (*p*-series, *p* = 2), we have by the LCT that** $\sum_{n=1}^{\infty}\left(\frac{n}{n^3-n+1}\right)$ **converges.**

2. Consider the series $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{7/4}}$. Complete the boxes and blanks.

a. $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{7/4}}$ will ______________
{[✓ converge], diverge}

b. Complete the box: We can use the Direct Comparison Test with $\sum_{n=1}^{\infty} \boxed{\frac{1}{n^{7/4}}}$ to validate our claim in part **a**.

c. Assuming the contents of the dashed box are same as the *n*th term of your series in part **b**, which choice is true?

Circle one and complete the box: [✓ **i.**] $\frac{\sin^2 n}{n^{7/4}} \leq \boxed{\frac{1}{n^{7/4}}}$ **ii.** $\boxed{} \leq \frac{\sin^2 n}{n^{7/4}}$

d. Complete, assuming the series in tbe dashed box below is what you wrote in the box in part **b**.

$\sum_{n=1}^{\infty} \boxed{\frac{1}{n^{7/4}}}$ will __________ because ***p*-series with $p = 7/4 > 1$**
{[✓ converge], diverge}
(Give a reason for your answer such as harmonic series, [✓ *p*-series with $p > 1$], *p*-series with $p < 1$, geometric series, *n*th Term Test for Divergence)

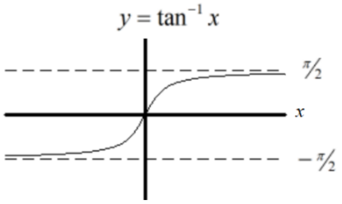
3. Consider the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{3^n}$. Complete the boxes and blanks.

a. $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{3^n}$ will ______________
{[✓ converge], diverge}

b. Complete the box: We can use the Direct Comparison Test with $\sum_{n=1}^{\infty} \boxed{\frac{\pi/2}{3^n}}$ to validate our claim in part **a**.

$\pi/2$ could be replaced by any higher value, i.e., $\frac{2}{3^n}$, since $\pi/2$ is the least upper bound of $\tan^{-1} n$.

$y = \tan^{-1} x$



c. Assuming the contents of the box are same as the *n*th term of your series in part **b**, which choice is true?

Circle one and complete the box: **i.** $\frac{\tan^{-1} n}{3^n} \leq \boxed{\frac{\pi/2}{3^n}}$ **ii.** $\boxed{} \leq \frac{\tan^{-1} n}{3^n}$

d. Complete, assuming the series in tbe box below is what you wrote in the box in part **b**.

$\sum_{n=1}^{\infty} \boxed{\frac{\pi/2}{3^n}}$ will ______________ because **Geometric series with $r = 1/3$**
{[✓ converge], diverge}
(Give a reason for your answer such as harmonic series, *p*-series with $p > 1$, *p*-series with $p < 1$, [✓ geometric series], *n*th Term Test for Divergence)

4. Consider the series $\sum_{n=1}^{\infty} \frac{n}{2n^2 - \cos^2 n}$. Complete the boxes and blanks.

$$2n^2 - \cos^2 n \le 2n^2$$

$$\frac{1}{2n^2} \le \frac{1}{2n^2 - \cos^2 n}$$

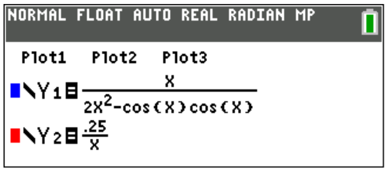
$$\frac{n}{2n^2} \le \frac{n}{2n^2 - \cos^2 n}$$

$$\frac{1}{2n} = \frac{1/2}{n} \le \frac{n}{2n^2 - \cos^2 n}$$

a. $\sum_{n=1}^{\infty} \frac{n}{2n^2 - \cos^2 n}$ will ________________
{converge, [✓ diverge]}

1/2 could be replaced by any lower value such as 1/4, i.e. $\frac{1/4}{n}$

since we want a function below $\frac{n}{2n^2 - \cos^2 n}$.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  Plot1   Plot2   Plot3
■\Y1■ X/(2X²-cos(X)cos(X))
■\Y2■ .25/X
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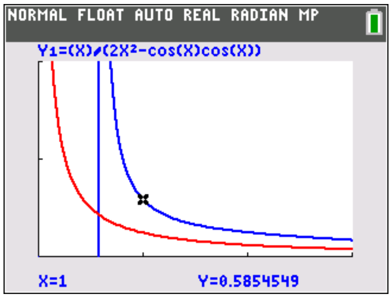
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Y1=(X)/(2X²-cos(X)cos(X))
X=1              Y=0.5854549
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b. Complete the box: We can use the Direct Comparison Test with $\sum_{n=1}^{\infty} \boxed{\frac{1/4}{n}}$ to validate our claim in part **a**.

c. Assuming the contents of the dashed box are same as the *n*th term of your series in part **b**, which choice is true?

Circle one and complete the box: **i.** $\frac{n}{2n^2 - \cos^2 n} \le \boxed{}$ **ii.** $\boxed{\frac{1/4}{n}} \le \frac{n}{2n^2 - \cos^2 n}$

d. Complete, assuming the series in tbe dashed box below is what you wrote in the box in part **b**.

$\sum_{n=1}^{\infty} \boxed{\frac{1/4}{n}}$ will ____________ because **Harmonic Series**
{converge, [✓ diverge]}

(Give a reason for your answer such as [✓ harmonic series], *p*-series with $p > 1$, *p*-series with $p < 1$, geometric series, *n*th Term Test for Divergence)

5. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{6n^2 + n + 7}{n^5 + 2n}$. Complete the boxes and blanks.

a We can use the Limit Comparison Test with $b_n = \boxed{\frac{1}{n^3}}$ to show $\sum_{n=1}^{\infty} \frac{6n^2 + n + 7}{n^5 + 2n}$ will ________________
{[✓ converge], diverge}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ________________ because ***p*-series with *p* = 3**
{[✓ converge], diverge}

(Give a reason for your answer such as harmonic series, [✓ *p*-series with $p > 1$], *p*-series with $p < 1$, geometric series, *n*th Term Test for Divergence)

The imit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{6}$.

$$\lim_{n\to\infty} a_n \cdot \frac{1}{b_n} = \lim_{n\to\infty} \frac{6n^2 + n + 7}{n^5 + 2n} \cdot \frac{n^3}{1} = \lim_{n\to\infty} \frac{6n^5}{n^5} = 6$$

6. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{5n^5 + 8n^2}{\sqrt{n^{12} + 8n^2}}$. Complete the boxes and blanks.

a. We can use the Limit Comparison Test with $b_n = \boxed{\frac{1}{n}}$ to show $\sum_{n=1}^{\infty} \frac{5n^5 + 8n^2}{\sqrt{n^{12} + 8n^2}}$ will ________ {converge, [✓ diverge]}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ________ {converge, [✓ diverge]} because **Harmonic Series**

(Give a reason for your answer such [✓ as harmonic series], p-series with $p > 1$, p-series with $p < 1$, geometric series, nth Term Test for Divergence.)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{5}$.

$$\lim_{n\to\infty} a_n \cdot \frac{1}{b_n} = \lim_{n\to\infty} \frac{5n^5 + + 8n^2}{\sqrt{n^{12} + 8n^2}} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{5n^6}{n^6} = \mathbf{5}$$

7. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{7}{\sqrt{n^3 + 4n + 16}}$. Complete the boxes and blanks.

a. We can use the Limit Comparison Test with $b_n = \boxed{\frac{1}{n^{3/2}}}$ to show $\sum_{n=1}^{\infty} a_n$ will ________ {[✓ converge], diverge}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ________ {[✓ converge], diverge} because ***p*-series with *p* = 3/2**

(Give a reason for your answer such as harmonic series, [✓ p-series with $p > 1$], p-series with $p < 1$, geometric series, nth Term Test for Divergence)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{7}$.

$$\lim_{n\to\infty} a_n \cdot \frac{1}{b_n} = \lim_{n\to\infty} \frac{7}{\sqrt{n^3 + 4n + 16}} \cdot \frac{n^{3/2}}{1} = \mathbf{7}$$

8. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{8}{2^n - 1}$. Complete the boxes and blanks.

a. We can use the Limit Comparison Test with $b_n = \boxed{\frac{1}{2^n} = \left(\frac{1}{2}\right)^n}$ to show $\sum_{n=1}^{\infty} a_n$ will ________ {[✓ converge], diverge}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ________ {[✓ converge], diverge} because **Geometric series with *r* = 1/2**

(Give a reason for your answer such as harmonic series, p-series with $p > 1$, p-series with $p < 1$, [✓ geometric series], nth Term Test for Divergence)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{8}$.

$$\lim_{n\to\infty} a_n \cdot \frac{1}{b_n} = \lim_{n\to\infty} \frac{8}{2^n - 1} \cdot \frac{2^n}{1} = \lim_{n\to\infty} \frac{8 \cdot 2^n}{2^n} = \mathbf{8}$$

9. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{5(n-1)(n+4)(n+3)(n+2)}{\left(n^3 + 3n\right)(2n+1)^2}$. Complete the boxes and blanks.

a. We can use the Limit Comparison Test with $b_n = \boxed{\frac{1}{n}}$ to show $\sum_{n=1}^{\infty} a_n$ will ________ {converge, [✓ diverge]}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ________ {converge, [✓ diverge]} because **Harmonic Series**

(Give a reason for your answer such [✓ as harmonic series], p-series with $p > 1$, p-series with $p < 1$, geometric series, nth Term Test for Divergence)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{\frac{5}{4}}$.

$$\lim_{n\to\infty} a_n \cdot \frac{1}{b_n} = \lim_{n\to\infty} \frac{5\left(n^4 + \text{terms of lower degree}\right)}{\left(n^3 + 3n\right)\left(4n^2 + \text{terms of lower degree}\right)} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{5n^5}{4n^5} = \frac{5}{4}$$

10. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{9e^{n}+4}{2e^{5n}-2}$. Complete the boxes and blanks.

a. We can use the Limit Comparison Test with $b_n = \boxed{\frac{1}{e^{4n}} = \left(\frac{1}{e^{4}}\right)^{n}}$ to show $\sum_{n=1}^{\infty} a_n$ will ______
{[✓ converge], diverge}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ______ because **Geometric Series with $r = e^{-4}$ (You could also use Integral Test)**
{[✓ converge], diverge}
(Give a reason for your answer such as harmonic series, p-series with $p > 1$, p-series with $p < 1$, [✓ geometric series], nth Term Test for Divergence)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{\frac{9}{2}}$

$$\lim_{n\to\infty} a_n \cdot \frac{1}{b_n} = \lim_{n\to\infty} \frac{9e^{n}+4}{2e^{5n}-2} \cdot \frac{e^{4n}}{1} = \lim_{n\to\infty} \frac{9e^{5n}+4e^{4n}}{2e^{5n}-2} = \lim_{n\to\infty} \frac{9e^{5n}}{2e^{5n}} = \frac{9}{2}$$