Practice Questions from 10.7-10.8 and 11.1-11.2

1. The Ratio Test and Root Test are based on the properties of convergence of
A. a p-series, $p\neq 1$. B. the harmonic series C. the alternating series D. a television series E. the world series F. a geometric series

2. Which of these will help you determine if the series $\sum_{n=0}^{\infty} 2e^n$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
D. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
E. integral test F. ratio test G. nth Term Test for Divergence

3. Which of these will help you determine if the series $\sum_{n=0}^{\infty} e^{-2n}$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
D. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
E. integral test F. ratio test G. nth Term Test for Divergence

4. Which of these will help you determine if the series $\sum_{n=1}^{\infty} \left(\frac{(-1)^{n+1}}{n^2} \right)$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
D. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
E. ratio test F. nth Term Test for Divergence

5. Which of these will help you determine if the series $\sum_{n=1}^{\infty} \left(\frac{(-1)^{n+1}}{\sqrt{n}} \right)$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
D. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
E. ratio test F. nth Term Test for Divergence

6. Which of these will help you determine if the series $\sum_{n=1}^{\infty} \left(\frac{n+2}{n!} \right)$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
D. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
E. integral test F. ratio test G. nth Term Test for Divergence

7. Use the Ratio Test for each.

a. The series $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$ will ________ {converge, diverge} by the Ratio Test because $\lim_{n\to\infty} \boxed{\frac{2}{n+1}} = \boxed{0}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = \frac{\cancel{2^n}\cdot 2\cdot \cancel{n!}}{(n+1)\cancel{n!}\,\cancel{2^n}} =$$

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

b. The series $\sum_{n=1}^{\infty} \frac{4^n}{n^{800}}$ will ________ {converge, diverge} by the Ratio Test because $\lim_{n\to\infty} \boxed{\frac{4n^{800}}{(n+1)^{800}}} = \boxed{4}$.

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{4^{n+1}}{(n+1)^{800}} \cdot \frac{n^{800}}{4^n} = \frac{4\cdot\cancel{4^n}\cdot n^{800}}{\cancel{4^n}\cdot(n+1)^{800}}$$

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

8. Use the Root Test for each.

a. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{8n^4}{7n^4+n+5}\right)^n$ will ________ {converge, diverge} by the Root Test because $\lim_{n\to\infty}$ $\boxed{\frac{8n^4}{7n^4+n+5}}$ = $\boxed{8/7}$

$|a_n|^{1/n} = \left(\frac{8n^4}{7n^4+n+5}\right)^{n/n} =$

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

b. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{2n}{3n+2}\right)^n$ will ________ {converge, diverge} by the Root Test because $\lim_{n\to\infty}$ $\boxed{\frac{2n}{3n+2}}$ = $\boxed{2/3}$.

$|a_n|^{1/n} = \left(\frac{2n}{3n+2}\right)^{n/n} =$

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

b. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n+1}{n}\right)^{n^2}$ will ________ {converge, diverge} by the Root Test because $\lim_{n\to\infty}$ $\boxed{\left(1+\frac{1}{n}\right)^n}$ = $\boxed{e}$.

$|a_n|^{1/n} = \left(\frac{n+1}{n}\right)^{n^2/n} = \left(1+\frac{1}{n}\right)^n \to e$

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

9. Consider the series $\sum_{n=1}^{\infty}\left(1+\frac{a}{n}\right)^{18n}$ for some real number **a**.

a. The series will ________ {converge, diverge}.

b. Circle the best answer to determine part **a**.
A. It is a *p*-series. B. It is a geometric series C. Use the Ratio Test D. Use the Root Test E. Use the *n*th Term Test for Divergence

c. Explain more fully below how part **b** justifies part **a**.

$\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{18n} = \lim_{n\to\infty}\left(\left(1+\frac{a}{n}\right)^n\right)^{18} = e^{a\cdot 18} \neq 0$

10. Answer the following for the power series $\sum c_n(x-a)^n$. Complete the blanks.

a. The power series $\sum c_n(x-a)^n$ is centered at the value $x =$ __a__.

b. Suppose the interval of convergence is **all real numbers**. Then the radius of convergence is $R =$ __∞__.

c. Suppose the interval of convergence is only **the value** $x = a$. Then the radius of convergence is $R =$ __0__.

d. Suppose the interval of convergence is $|x-a| < b$, i.e. $a-b < x < a+b$. Then the radius of convergence is $R =$ __b__.

11. The interval of convergence of $\sum_{n=1}^{\infty}\left(\frac{x-4}{2}\right)^n$ is $\boxed{2} < x < \boxed{6}$. Show work below.

Hint: It is a geometric series.

$-1 < \frac{x-4}{2} < 1$

$-2 < x-4 < 2$

$4-2 < x < 4+2$

$2 < x < 6$

12. Report the interval of convergence of $\sum_{n=0}^{\infty} n!x^{5n}$. Select one. Use ratio test to find which values of x make it converge.

A. $-1<x<1$ B. $-\frac{1}{\sqrt{5}}<x<\frac{1}{\sqrt{5}}$ C. $-\sqrt[5]{5}<x<\sqrt[5]{5}$ D. $x=0$ E. $-\frac{1}{5}<x<\frac{1}{5}$ F. $-\infty<x<\infty$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!\,x^{5(n+1)}}{n!\quad x^{5n}}\right| = \lim_{n\to\infty}\left(\frac{\cancel{n!}(n+1)}{\cancel{n!}}\left|\frac{\cancel{x^{5n}}x^5}{\cancel{x^{5n}}}\right|\right) = \lim_{n\to\infty}(n+1)\cdot|x^5| = \infty > 1$$ for all x.

13. The interval of convergence of $\sum_{n=1}^{\infty}\frac{x^{3n}}{n!}$ is $\boxed{-\infty} < x < \boxed{\infty}$. Show work below.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{3n+3}}{(n+1)!}\cdot\frac{n!}{x^{3n}}\right| = \lim_{n\to\infty}\left(\frac{\cancel{n!}}{\cancel{n!}(n+1)}\left|\frac{\cancel{x^{3n}}x^3}{\cancel{x^{3n}}}\right|\right) = \lim_{n\to\infty}\frac{1}{n+1}\cdot|x^3| = 0 < 1$$ for all x.

14. Consider $\sum_{n=1}^{\infty}\frac{(3x)^n}{n}$

a. The radius of convergence is $R =$ 1/3. Show work below.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(3x)^{n+1}}{(n+1)}\cdot\frac{n}{(3x)^n}\right| = \lim_{n\to\infty}\frac{n}{n+1}\cdot|3x| = \lim_{n\to\infty}|3x| < 1$$ for

$-1 < 3x < 1$

$-1/3 < x < 1/3$

before checking endpoints

b. If x is equal to the **left endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty}\frac{(3x)^n}{n}$ will ______. {converge, diverge}

If $x = -\frac{1}{3}$, we have $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

c. If x is equal to the **right endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty}\frac{(3x)^n}{n}$ will ______. {converge, diverge}

If $x = 1/3$, we have $\sum_{n=1}^{\infty}\frac{1}{n}$

d. State the **reasons** which justify your claims about the endpoints in parts **b** and **c**.

part b: Alternating Harmonic Series (or AST)

part c: Harmonic Series

So the interval of convergence is

$-\frac{1}{3} \leq x$ 1/3

15. Consider $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$

a. The radius of convergence is $R =$ 1. Show work below.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{n+12}}{(n+1)^2}\cdot\frac{n^2}{x^{n+11}}\right| = \lim_{n\to\infty}\left(\frac{n^2}{(n+1)^2}\left|\frac{\cancel{x^n}\,x^{12}}{\cancel{x^n}\cdot x^{11}}\right|\right)$$

$$= \lim_{n\to\infty}\frac{n^2}{(n+1)^2}\cdot\lim_{n\to\infty}|x| = 1\cdot\lim_{n\to\infty}|x| < 1 \text{ if}$$ $|x|<1$ or $-1<x<1$ before checking endpoints

b. If x is equal to the **left endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$ will ______. {converge, diverge}

If $x=-1$, we have $\sum_{n=1}^{\infty}(-1)^n(-1)^1\cdot\left(\frac{(-1)^n(-1)^{11}}{n^2}\right) = \sum_{n=1}^{\infty}\frac{1}{n^2}$

c. If x is equal to the **right endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$ will ______. {converge, diverge}

If $x=1$, we have $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{1}{n^2}$

d. State the **reasons** which justify your claims about the endpoints in parts **b** and **c**.

part b: p-series $p=2$

part c: A.S.T

Fun Facts:

For all x we have $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ $\quad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $\quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

For $-1 < x < 1$ we have $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$

For $-1 < x \le 1$ we have $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ $\quad$ For $-1 \le x \le 1$ we have $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

16. Complete: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots = \boxed{\ln 2}$. The name of this series is called the Alternating Harmonic series.

TIP: Use a Fun Fact above.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

17. Complete: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots = \boxed{\infty}$. The name of this series is called the Harmonic series.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

18. a. In sigma notation the series $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots = \sum_{\boxed{n=1}}^{\infty} \boxed{-\frac{x^n}{n}}$

b. Use one of the Fun Facts above to determine what function $f(x)$ the series $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots$ approximates.

$f(x) = \boxed{\ln(1-x)}$. The radius of convergence is $R = \boxed{1}$

Simplified please.

c. Write the first four terms of the series in expanded form if $x = -1$. $\boxed{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}} + \ldots$

The left endpoint $x = -1$ ______ in the interval of convergence. Explain your answer.
{is, is not}

Reason: Question 16. It is Alt. Harm. Series

Simplified please.

d. Write the first four terms of the series in expanded form if $x = 1$. $\boxed{-1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4}} - \ldots$

The right endpoint $x = 1$ ______ in the interval of convergence. Explain your answer.
{is, is not}

Reason: Question 17. It is the harmonic series

19. Consider function $f(x) = 10\tan^{-1}(2w)$. Write the first **four** terms of the series.

Simplified please.

$10\tan^{-1}(2w) = \boxed{20w - \frac{80w^3}{3} + \frac{64w^5}{5} - \frac{128w^7}{7}} + \ldots$

use $\tan^{-1}(\square) = \square - \frac{\square^3}{3} + \frac{\square^5}{5} - \frac{\square^7}{7}$ $\quad \square = 2w$

so $\tan^{-1}(2w) = 2w - \frac{8w^3}{3} + \frac{32w^5}{5} - \frac{128w^7}{7}$

and now multiply each term by 10

20. Consider function $f(w) = \sin(w^2)$. Write the first **four** terms of the series.

Simplified please.

$\sin(w^2) = \boxed{w^2 - \frac{w^6}{3!} + \frac{w^{10}}{5!} - \frac{w^{14}}{7!}} + \ldots$

21. Consider function $f(w) = e^{-w}$. Write the first **four** terms of the series.

Simplified please.

$e^{-w} = \boxed{1 - w + \frac{w^2}{2!} - \frac{w^3}{3!}} + \ldots$

use $e^{\square} = 1 + (\boxed{-w}) + \frac{(\boxed{-w})^2}{2!} + \frac{(\boxed{-w})^3}{3!} + \ldots$ $\quad \square = -w$

22. The term-by-term derivative of $f(x) = \sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ is the power series below.

a. Write the first four nonzero terms of the series for $f'(x)$.

Simplified please

$f'(x) = \boxed{5 + 10x + 15x^2 + 20x^3} + \ldots$

b. The radius of convergence of $f'(x)$ is $R =$ 1 This is a geometric series with $r = x$ and $a = 5$ so it converges if $|x| < 1$

c. If x is equal to the **left endpoint** of the interval of convergence, the series for $f'(x)$ will ________.

$x = -1 \Rightarrow 5 - 10 + 15 + 20 - \cdots$ diverges by oscillation {converge, diverge}

d. If x is equal to the **right endpoint** of the interval of convergence, the series for $f'(x)$ will ________.

$x = 1 \Rightarrow 5 + 10 + 15 + 20 + \cdots = \infty$ {converge, diverge}

e. Write the series for $f'(x)$ in sigma notation.

$f'(x) = \sum_{n=\boxed{1}}^{\infty} \left(\boxed{5nx^{n-1}}\right)$ Take the derivative with respect to x of $5x^n$ to get $5nx^{n-1}$

f. When x is in the interval of convergence, we can write the series for $f'(x)$ as what rational function?

$f'(x) = \dfrac{\boxed{5}}{\boxed{(1-x)^2}}$

$f(x) = 5 + 5x + 5x^2 + 5x^3 + \cdots = \dfrac{5}{1-x}$ on $-1 < x < 1$

By Geometric Series

Differentiate the right hand side

$$\frac{d}{dx} \cdot \frac{5}{(1-x)} = \frac{d}{dx} \cdot 5(1-x)^{-1}$$

$$= 5 \frac{d}{dx}(1-x)^{-1}$$

$$= 5 \cdot -(1-x)^{-2} \cdot \frac{d}{dx}(-x)$$

$$= 5 \cdot (-1) \cdot \frac{1}{(1-x)^2} \cdot (-1)$$

$$= \frac{5}{(1-x)^2}$$