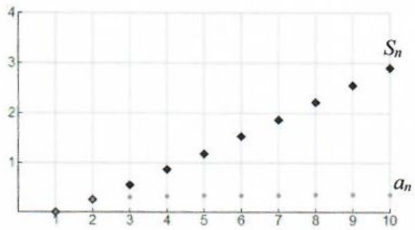
Practice Questions from Section 10.4

1. For what values of p does $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converge? $p > 1$ For what values of r does $\sum_{k=1}^{\infty} ar^{k-1}$ converge? $|r| < 1$

2. Shown to the right is a plot of the terms a_n and the nth partial sums S_n for the series $\sum_{k=1}^{\infty}\left(1-\frac{1}{k}\right)^k$.



a. Circle the correct choice: We know that $\lim_{k\to\infty}\left(1-\frac{1}{k}\right)^k$ ____ equal 0.
{does, does not}

This means by the nth Term Test for Divergence that

the series $\sum_{k=1}^{\infty}\left(1-\frac{1}{k}\right)^k$ ____.
{converges, diverges, is nonconclusive}

b. The plot shows $\lim_{k\to\infty}\left(1-\frac{1}{k}\right)^k \approx 0.36787944117144232159552377016146086744581113103176783450783680169746149574489980 34$

$\lim_{k\to\infty}\left(1-\frac{1}{k}\right)^k = \lim_{k\to\infty}\left(1+\frac{-1}{k}\right)^k$

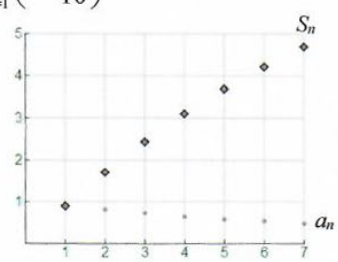
What is the exact value of this number? $\lim_{k\to\infty}\left(1-\frac{1}{k}\right)^k = \boxed{e^{-1}}$ Write in the box an **exact** number or DNE or ∞ or $-\infty$.

You can check with a grapher:

e^{-1}	
	0.3678794412

3. Shown to the right is a plot of the terms a_n and the nth partial sums S_n for the series $\sum_{k=1}^{\infty}\left(1-\frac{1}{10}\right)^k$.

$1 - 1/10 = 9/10$



a. Circle the correct choice: We know that $\lim_{k\to\infty}\left(1-\frac{1}{10}\right)^k$ ____ equal 0,
{does, does not}

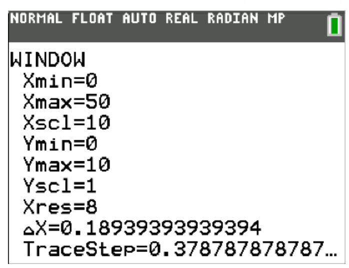
This means by the nth Term Test for Divergence that

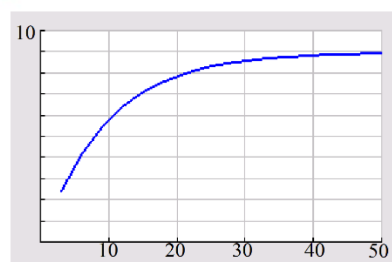
the series $\sum_{k=1}^{\infty}\left(1-\frac{1}{10}\right)^k$ ____.
{converges, diverges, is nonconclusive}

b. The series $\sum_{k=1}^{\infty}\left(1-\frac{1}{10}\right)^k = \boxed{9}$ Write in the box an **exact** number or DNE or ∞ or $-\infty$.

$\hookrightarrow$ Geometric with $r=\frac{9}{10}$ and $a=\frac{9}{10} \Rightarrow \frac{a}{1-r} = \frac{\frac{9}{10}}{\frac{1}{10}} = 9$

You can check with a grapher if you need convinced.





4. Suppose $\sum_{k=1}^{\infty} a_k$ is any series. Circle True or False. If False, give a counterexample which shows it is False. If True, leave blank.

a. True False If $\lim_{k\to\infty} a_k = 0$, then $\sum_{k=1}^{\infty} a_k$ converges. Harmonic Series or any p-series with $p \le 1$

b. True False If the limit of the sequence of partial sums S_n exists, i.e., $\lim_{n\to\infty} S_n = \lim_{n\to\infty}\sum_{k=1}^{n} a_k = L < \infty$, then $\sum_{k=1}^{\infty} a_k = L$, where L is some finite number.

This essentially is what it means for a series to converge.

c. True False If the limit of the sequence a_k exists, i.e. $\lim_{k\to\infty} a_k = L$, then the series $\sum_{k=1}^{\infty} a_k$ converges.

Use the series in Question 2 or any a_k where $\lim_{k\to\infty} a_k = L \neq 0$

d. True False If $\sum_{k=1}^{\infty} a_k$ converges, then the sequence of partial sums S_n approaches 0, i.e, $\lim_{n\to\infty} S_n = \lim_{n\to\infty}\sum_{k=1}^{n} a_k = 0$.

Use the series in Question 3

or any geometric series with $|r|<1$ and $a \neq 0$

5. Determine whether $\sum_{n=1}^{\infty} \frac{560n^4 \cdot 4^n + 11^n}{n^4 11^n}$ converges or diverges by answering the questions below.

a. Circle the correct answer below.

A. The series is the sum of a geometric series with $|r| < 1$ and the harmonic series.
B. The series is the sum of a geometric series with $|r| < 1$ and a p-series with $p < 1$.
C. The series is the sum of a geometric series with $|r| < 1$ and a p-series with $p > 1$.
D. The series is the sum of a geometric series with $|r| > 1$ and the harmonic series.
E. The series is the sum of a geometric series with $|r| > 1$ and a p-series with $p < 1$.
F. The series is the sum of a geometric series with $|r| > 1$ and a p-series with $p > 1$.

b. Write $\sum_{n=1}^{\infty} \frac{560n^4 \cdot 4^n + 11^n}{n^4 11^n}$ as the sum of a geometric series and a p-series (or harmonic series).

$$= \sum_{n=1}^{\infty} 560\left(\frac{4}{11}\right)^n + \sum_{n=1}^{\infty} \frac{1}{n^4}$$

c. Circle the correct choice and fill in the blanks: The geometric series has $r = \underline{\frac{4}{11}}$ and will ______ {converge, diverge}

and the p-series (or harmonic series) has $p = \underline{4}$ and will ______ {converge, diverge}.

Thus the series $\sum_{n=1}^{\infty} \frac{560n^4 \cdot 4^n + 11^n}{n^4 11^n}$ will ______ {converge, diverge}.

6. Because $f(x) = \frac{24}{1+x^2}$ is ______ {increasing, decreasing} for $x \geq 1$ we can use the Integral Test to show $\sum_{n=1}^{\infty} \frac{24}{1+n^2}$ converges or diverges.

a. The series will ______ {converge, diverge} because $\int_1^{\infty} \frac{24}{1+x^2}\,dx = \boxed{6\pi}$. Write in the box an **exact** number or DNE or ∞ or $-\infty$.

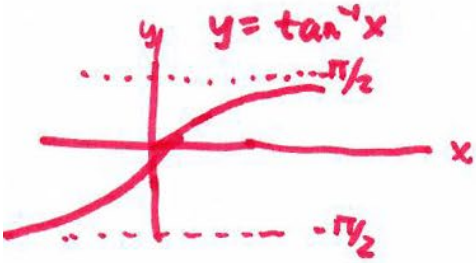
b. Show work below using correct limit notation:

$$\int_1^{\infty} \frac{24}{1+x^2}\,dx = 24 \int_1^{\infty} \frac{24}{1+x^2}\,dx = 24 \lim_{b\to\infty} \int_1^b \frac{1}{1+x^2}\,dx$$

$$= 24 \lim_{b\to\infty} \tan^{-1} x \Big|_1^b$$

$$= 24 \lim_{b\to\infty} \left(\tan^{-1} b - \tan^{-1} 1\right)$$

$$= 24 \cdot \left(\frac{\pi}{2} - \frac{\pi}{4}\right) = 6\pi \approx 18.85$$



c. Shown below is a plot of a_n and S_n for the series $\sum_{n=1}^{\infty} \frac{24}{1+n^2}$, as well as the area representing $\int_1^{\infty} \frac{24}{1+x^2}\,dx$.

Circle the best choice:

From the graphs we expect that $\sum_{n=1}^{\infty} \frac{24}{1+n^2}$ $\underline{>}$ {=, <, >} $\int_1^{\infty} \frac{24}{1+x^2}\,dx$

24 ish

18.85