Practice Questions from HW 17-18 (Section 10.1-10.3) to prepare for Quiz 6.
Note: The actual quiz will be shorter.

1. Complete: $\sum_{k=0}^{\infty} 400(1.10)^k = \boxed{\infty}$ Write in the box an exact number or DNE or ∞ or −∞.

This is a geometric series with $r = 1.10 > 1$
It is the sum of the series in Question 9, climbing without bound to infinity.

2. Complete: $\sum_{k=0}^{\infty} \frac{282}{13^{k-1}} = \boxed{3971.5}$ Write in the box an exact number or DNE or ∞ or −∞.

If $\sum_{k=0}^{\infty} \frac{282}{13^{k-1}}$ were written as $\sum_{k=0}^{\infty} ar^k$, report a and r. $a = \underline{3666}$ $r = \underline{1/13}$.

$\frac{a}{1-r} = \frac{3666}{1-1/13} = \frac{3666}{12/13}$

$\frac{282}{13^{k-1}} = \frac{282}{13^k 13^{-1}} = \frac{282 \cdot 13}{13^k} = 3666 \cdot \left(\frac{1}{13}\right)^k$

$= 3666 \cdot \frac{13}{12}$
$= 3971.5$

3. The series $\sum_{k=0}^{\infty} ar^k$ converges to 5. If $a = 9.5$, what is the value of r? Complete: $\sum_{k=0}^{\infty} 9.5\left(\boxed{\frac{-9}{10}}\right)^k = 5$

Show work. $5 = \frac{a}{1-r} = \frac{9.5}{1-r} \Rightarrow 1-r = \frac{9.5}{5}$

$1-r = 1.9 \Rightarrow r = 1-1.9 = -0.9 = -\frac{9}{10}$

4. The series $\sum_{k=0}^{\infty} ar^k$ converges to 5. If $r = \frac{1}{25}$, what is the value of a? Complete: $\sum_{k=0}^{\infty} \boxed{}\left(\frac{1}{25}\right)^k = 5$

Show work.

$5 = \frac{a}{1-r} = \frac{a}{1-\frac{1}{25}} = \frac{a}{\frac{24}{25}}$ so $a = 5 \cdot \frac{24}{25} = \frac{24}{5}$

5. For what values of r does the series $\sum_{k=0}^{\infty} a(r)^k$ converge? $\underline{-1 < r < 1}$ or $|r| < 1$

6. Consider the function $f(x) = \sum_{k=0}^{\infty} 9\left(\frac{x-4}{2}\right)^k$

a. Evaluate $f(3)$. Show work. $f(3) = \boxed{6}$ Write in the box an exact number or DNE or ∞ or −∞.

$f(3) = \sum_{k=0}^{\infty} 9\left(-\frac{1}{2}\right)^k = \frac{a}{1-r}$ with $a = 9$, $r = -1/2$ $\Rightarrow \frac{9}{1-(-1/2)} = \frac{9}{\frac{3}{2}} = 9 \cdot \frac{2}{3} = 6$

b. For what values of x does $f(x)$ converge? Show work.

Solve $-1 < \frac{x-4}{2} < 1$ for x

Multiply all parts by 2: $-2 < x-4 < 2$
Add 4 to all parts : $-2+4 < x < 2+4$
$2 < x < 6$

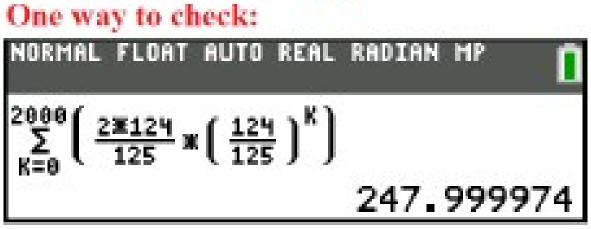
$\boxed{2} < x < \boxed{6}$

7. Complete: $\frac{2 \cdot 124}{125} + \frac{2 \cdot 124^2}{125^2} + \frac{2 \cdot 124^3}{125^3} + \ldots = \boxed{248}$ Write in the box an exact number or DNE or ∞ or −∞.

$a =$ first term $= \frac{2 \cdot 124}{125}$ and $r = \frac{124}{125}$ so $\frac{a}{1-r} = \frac{\frac{2 \cdot 124}{125}}{1 - \frac{124}{125}} = \frac{\frac{2 \cdot 124}{125}}{\frac{1}{125}} = \frac{2 \cdot 124 \cdot \frac{1}{\cancel{125}}}{1 \cdot \frac{1}{\cancel{125}}} = 2 \cdot 124 = \boxed{248}$

1 out

One way to check:



Another way:

8. Consider the sequence given by the recurrence relation $a_{n+1} = 0.95a_n + 8.2, \quad a_1 = 8.2$

a. The sequence converges to a limit L. Give the exact value of L. $L = \boxed{164}$

Since 0.95 is so close to 1, convergence is slow; the graph takes a very long time before it gets close to its horizontal asymptote.

b. Convergence occurs when $a_{n+1} = a_n$. Use this fact to rewrite the above recurrence relation into an equation that involves L.

Equation: $L = 0.95L + 8.2$

c. ***Solve*** the equation in part **b** to justify your claim in part **a**.

$L - 0.95L = 8.2$

$L(1 - 0.95) = 8.2$

$L = \dfrac{8.2}{1-0.95} = 164$

d. Complete the boxes below to write the next two terms of the series in long form.
Each subsequent term involves a numerical expression containing 0.95 and 8.2.

$8.2 + \boxed{8.2(0.95)} + \boxed{8.2(0.95)^2} + \ldots$

e. Without using sigma notation, write an expression that gives the nth partial sum of this series $S_n = \dfrac{\boxed{8.2}\left(1 - \boxed{.95}^{\boxed{n}}\right)}{\boxed{1 - 0.95}}$
i.e., the sum of the series of n terms.

f. Enter your expression from part **e** in your grapher and scroll a table to find the value of n for which the sum first surpasses 150.

X	Y1
1	8.2
2	15.99
3	23.391
⋮	⋮
47	149.28
48	150.02
49	150.72

The number of terms $n = \boxed{48}$

Check sum is 164:

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Y1 = 8.2(1-0.95^X) / (1-0.95)

TABLE SETUP
 TblStart=1000
 ΔTbl=500
Indpnt: Auto Ask
Depend: Auto Ask
```

X	Y1
1000	164
1500	164
2000	164
2500	164
3000	164
3500	164
4000	164
4500	164
5000	164
5500	164
6000	164

9. Once per year Richie Rich deposits an amount of \$400 in an account which pays 10% interest per year, compounded annually, with **additional deposits of \$400 continually made at the end of the year**.

If B_n is the balance in the account, in dollars, immediately after Richie makes the nth deposit, then we can write B_1 = \$400.

a. Complete the table to find the following. Report to the nearest \$0.01.

i) the balance, B_2, of the account on the day immediately after the second deposit.
ii) the balance, B_3, of the account on the day immediately after the third deposit.
iii) the balance, B_4, of the account on the day immediately after the fourth deposit.

n, # Deposits	B_n
1	\$400
2	840
3	1324
4	1856.40

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400
                                400
1.1Ans+400
                                840
1.1Ans+400
                               1324
1.1Ans+400
                             1856.4
```

b. Suppose Richie makes 422 deposits. Which is true about the sum B_{422}?
The balance, B_{422}, of the account on the day immediately after the 422nd deposit is exactly

A. $B_{422} = 400 \cdot 10^{422} + 400 \cdot 10^{421} + \ldots + 400 \cdot 10^2 + 400 \cdot 10 + 400$

B. $B_{422} = 400 \cdot 1.10^{423} + 400 \cdot 1.10^{422} + \ldots + 400 \cdot 1.10^2 + 400 \cdot 1.10 + 400$

C. $B_{422} = 400 \cdot 10^{423} + 400 \cdot 10^{422} + \ldots + 400 \cdot 10^2 + 400 \cdot 10 + 400$

D. $B_{422} = 400 \cdot 1.10^{422} + 400 \cdot 1.10^{421} + \ldots + 400 \cdot 1.10^2 + 400 \cdot 1.10 + 400$

E. $B_{422} = 400 \cdot 1.10^{421} + 400 \cdot 1.10^{420} + \ldots + 400 \cdot 1.10^2 + 400 \cdot 1.10 + 400$ ← we have 422 deposits of 400

F. $B_{422} = 400 \cdot 10^{421} + 400 \cdot 10^{420} + \ldots + 400 \cdot 10^2 + 400 \cdot 10 + 400$

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NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
Y1 = 400(1-1.10^X) / (1-1.10)
```

c. The balance, B_{422}, of the account on the day immediately after the 422nd deposit is approximately

A. $B_{422} \approx$ \$1291712354137103000000

B. $B_{422} \approx$ \$1067530871187688000000

C. $B_{422} \approx$ \$1174283958306457000000 ←

D. $B_{422} \approx$ \$1188774622351958700000

E. $B_{422} \approx$ \$1449066404550168000

F. The value of B_{422} can not be computed.

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NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl
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X	Y1
1	400
2	840
3	1324
4	1856.4
⋮	⋮
422	1.2E21
423	1.3E21
424	1.4E21
425	1.6E21
426	1.7E21
427	1.9E21
428	2.1E21
429	2.3E21
430	2.5E21

Y1=1.174283958307E21

calculate $\dfrac{400(1 - 1.10^{422})}{1 - 1.10}$

10. Consider the function $f(x) = \sum_{k=1}^{\infty} 100\left(\frac{-x}{10}\right)^{k+1}$

a. Write out the first four terms: $f(x) = \sum_{k=1}^{\infty} 100\left(\frac{-x}{10}\right)^{k+1} = \boxed{x^2} + \boxed{\frac{-x^3}{10}} + \boxed{\frac{x^4}{10^2}} + \boxed{\frac{-x^5}{10^3}} + \ldots$

b. Evaluate: $f(0) = \boxed{0}$ Write in the box an exact number or DNE or ∞ or $-\infty$.

This is $0 - 0 + 0 - 0 + \cdots$ or $\sum_{K=1}^{\infty} 100 \cdot 0^{K+1}$

c. Evaluate: $f(10) = \boxed{\text{DNE}}$ Write in the box an exact number or DNE or ∞ or $-\infty$.

This is $100 - 100 + 100 - 100 + \cdots$
which oscillates to 100 or 0 so DNE

d. Evaluate: $f(20) = \boxed{\text{DNE}}$ Write in the box an exact number or DNE or ∞ or $-\infty$.

This is a geometric series with $r = -2$
which oscillates to ∞ or $-\infty$ so DNE

e. For what values of x does $f(x)$ converge? Show work. $a = x^2$, $r = \frac{-x}{10}$

$-1 < \frac{-x}{10} < 1$

Multiply both sides by -10: $10 > x > -10$
or $-10 < x < 10$

$\boxed{-10} < x < \boxed{10}$

f. Find the sum, assuming x is in the interval in part **e**. Simplify.

$\frac{a}{1-r} = \frac{x^2}{1-\left(-\frac{x}{10}\right)} = \frac{(x^2) \cdot 10}{\left(1+\frac{x}{10}\right) \cdot 10} =$

$f(x) = \sum_{k=1}^{\infty} 100\left(\frac{-x}{10}\right)^{k+1} = \boxed{\frac{10x^2}{10+x}}$

11. Complete the boxes and evaluate each of the following series. If it diverges to ∞, then insert ∞ in the answer box.

a. $f(x) = \sum_{k=0}^{\infty} \frac{1380}{5^{2-k}} = 55.2 + 276 + 1380 + 6900 + \ldots$

i. $a = \boxed{55.2}$ $r = \boxed{5}$ $: \frac{276}{55.2} = \frac{1380}{276} = \frac{6900}{1380} = 5$

ii. $f(x) = \sum_{k=0}^{\infty} \frac{1380}{5^{2-k}} = 55.2 + 276 + 1380 + 6900 + \ldots = \boxed{\infty}$

iii. Give a reason for your claim in part **ii.** that does not have anything to do with technology.

geometric series with $r = 5$ which is >1; terms increase to ∞

b. $f(x) = \sum_{k=0}^{\infty} \frac{1380}{5^{k-2}} = 34500 + 6900 + 1380 + 276 + 55.2 + \ldots$

i. $a = \boxed{34500}$ $r = \boxed{0.2}$ or $\frac{1}{5}$

ii. $f(x) = \sum_{k=0}^{\infty} \frac{1380}{5^{k-2}} = 34500 + 6900 + 1380 + 276 + 55.2 + \ldots = \boxed{43125}$

iii. Give a reason for your claim in part **ii.** that does not have anything to do with technology.

$\frac{a}{1-r} = \frac{34500}{1-0.2} = \frac{34500}{0.8} = 43125$ since $r = \frac{1}{5}$ which is < 1

12. Consider the function $f(x)=\sum_{k=1}^{\infty} e^{-kx}$

a. Write out the first four terms, exactly: $f(x)=\sum_{k=1}^{\infty} e^{-kx} = \boxed{e^{-x}} + \boxed{e^{-2x}} + \boxed{e^{-3x}} + \boxed{e^{-4x}} + \ldots$

b. Evaluate: $f(0)=\boxed{\infty}$ Write in the box, an exact number or DNE or ∞ or $-\infty$.

this is $e^0+e^0+e^0+\cdots = 1+1+1+\cdots$ which increases off to infinity

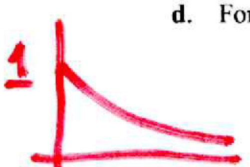
c. Evaluate: $f(6)=\boxed{\frac{1}{e^6-1}}$ Write in the box, an exact number or DNE or ∞ or $-\infty$.

$\sum_{k=1}^{\infty} e^{-6k} = e^{-6}+(e^{-6})^2+(e^{-6})^3+\cdots$

$a=e^{-6},\ r=e^{-6}$

$\frac{a}{1-r}=\frac{e^{-6}}{1-e^{-6}}\cdot\frac{e^6}{e^6}=$

d. For what values of x does $f(x)$ converge? Show work.



$\boxed{0} < x < \boxed{\infty}$

$r=e^{-x}<1$ for $x>0$. (e^{-x} is never negative.)

e. Find the exact sum, assuming x is in the interval in part **d**. $f(x)=\sum_{k=1}^{\infty} e^{-kx} = \boxed{\frac{1}{e^x-1}}$

$a=e^{-x},\ r=e^{-x}$

$\frac{a}{1-r}=\frac{e^{-x}}{1-e^{-x}}\cdot\frac{e^x}{e^x}=\frac{1}{e^x-1}$

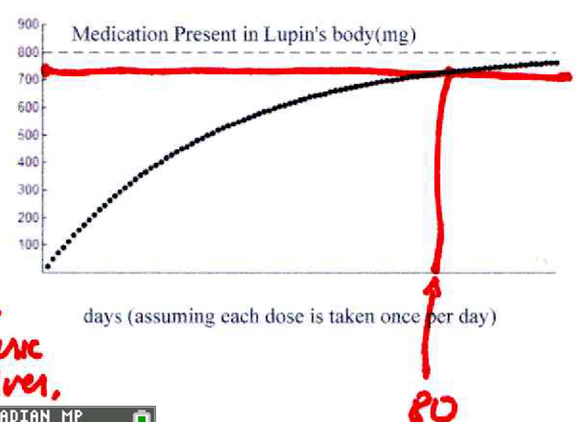
13. Professor Snape needs to create a potion for Remus Lupin to address the negative effects of his lycanthropy. Unfortunately, this medication takes a very long time to stabilize. Snape wants the stabilization level to eventually be 800 mg. For this to happen, Lupin must take the potion once per day in perpetuity. Lupin's body will eliminate only 3% of the medication between each dose. Answer the questions below. $r=0.97$

a. What dosage should Professor Snape prescribe so that the drug stabilization level will be 800 mg?

Lupin must take $\boxed{24}$ mg each day. Show your calculations. $\frac{a}{1-r}=800 \Rightarrow \frac{a}{1-.97}=800 \Rightarrow a=.03\cdot 800=$

b. Create a formula which gives the amount of medication that is present, in mg, in Lupin's body right after the xth dose of the amount prescribed in part **a.** $A(x)=\boxed{800-800(0.97)^x}$

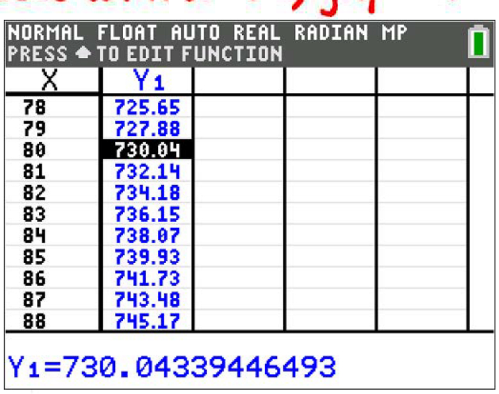
c. To the right is a graph of the formula in part **b**.

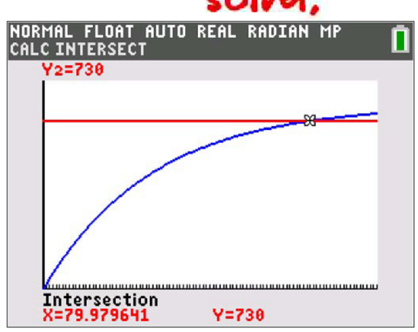


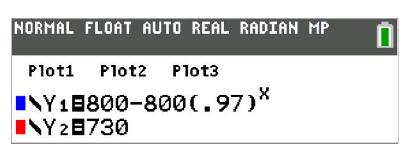
days (assuming each dose is taken once per day)

The drug will take effect when the medication level in Lupin's body is first within 730 mg. How many days of regular doses will it take for the drug to take effect? It will take $\boxed{80}$ days to reach a level of 730 mg, assuming Lupin takes one dose every day as prescribed. No work need be shown. Utilize your technology.

Solve with a table, graphical intersection, or a numeric solver.







14. Find the exact value of k for which $e^k + e^{2k} + e^{3k} + e^{4k} + \ldots = 99$

$e^k + e^{2k} + e^{3k} + e^{4k} + \ldots = e^k + (e^k)^2 + (e^k)^3 + (e^k)^4 + \ldots$

$a = e^k$, $r = e^k$ so $\dfrac{a}{1-r} = \dfrac{e^k}{1-e^k} = 99$

$$e^k = 99(1-e^k)$$
$$e^k = 99 - 99e^k$$
$$e^k + 99e^k = 99$$
$$100e^k = 99$$
$$e^k = 0.99$$
$$\ln e^k = \ln 0.99$$
$$k = \ln 0.99$$

Check: If $k = \ln 0.99$, then

$e^k + e^{2k} + e^{3k} + e^{4k} + \ldots = e^{\ln 0.99} + (e^{\ln 0.99})^2 + (e^{\ln 0.99})^3 + (e^{\ln 0.99})^4 + \ldots$

$= 0.99 + (0.99)^2 + (0.99)^3 + (0.99)^4 + \ldots$

So $a = 0.99$ and $r = 0.99$ and $0.99 + (0.99)^2 + (0.99)^3 + (0.99)^4 + \ldots = \dfrac{a}{1-r} = \dfrac{0.99}{1-0.99} = \dfrac{0.99}{0.01} = 99$

See next page.

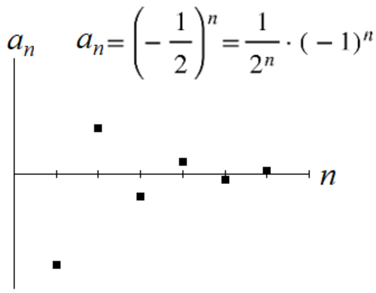
15. For what values of r does $a_n = r^n$ converge? For what values of r does $a_n = r^n$ diverge?

Think about the graphs of $y = r^n$ for various values of r, exploring representative samples.

For $|r| < 1$, then consider $r = -\dfrac{1}{2}$ or $r = \dfrac{1}{2}$.

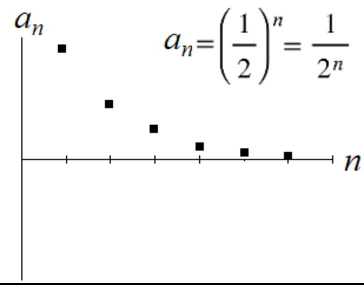
If $r = -\dfrac{1}{2}$ then $a_n = \left(-\dfrac{1}{2}\right)^n = \dfrac{1}{2^n} \cdot (-1)^n \;\to 0$ as $n \to \infty$.

So a_n **converges**.



If $r = \dfrac{1}{2}$ or $a_n = \left(\dfrac{1}{2}\right)^n = \dfrac{1}{2^n} \to 0$ as $n \to \infty$.

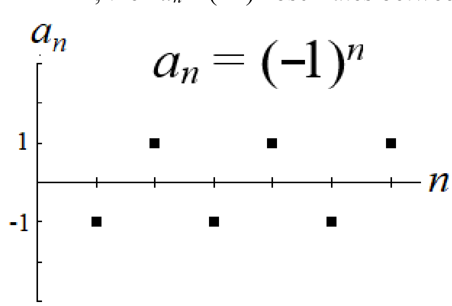
So a_n **converges**.



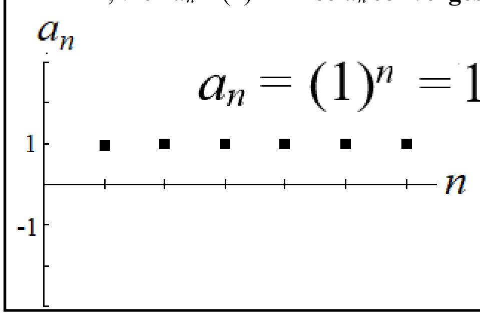
For $|r| > 1$, then we expect $a_n = r^n$ to diverge as $n \to \infty$. Graphs would confirm this if you wanted to make sketches.

Explore the case for $r = -1$ and for $r = 1$.

If $r = -1$, then $a_n = (-1)^n$ oscillates between -1 and 1 so a_n **diverges**.

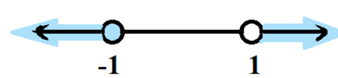


If $r = 1$, then $a_n = (1)^n = 1$ so a_n **converges**.



In conclusion, $a_n = r^n$ converges for $|r| < 1, r = 1$.
We can also write this as a compound inequality $-1 < r \le 1$.

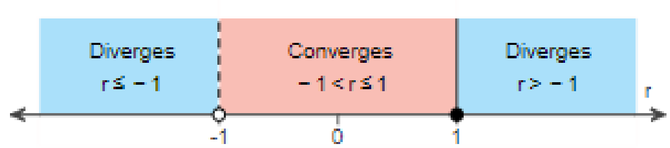
Thus $a_n = r^n$ diverges for $|r| > 1, \; r = -1$.
We can also write this as the two inequalities $r \le -1, r > 1$.



If r is a real number, then the following is true.

$$\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } |r| < 1 \\ 1 & \text{if } r = 1 \\ \text{does not exist} & \text{if } r \le -1 \text{ or } r > 1 \end{cases}$$

If $r > 0$, then $\{r^n\}$ is a monotonic sequence. If $r < 0$, then $\{r^n\}$ oscillates.

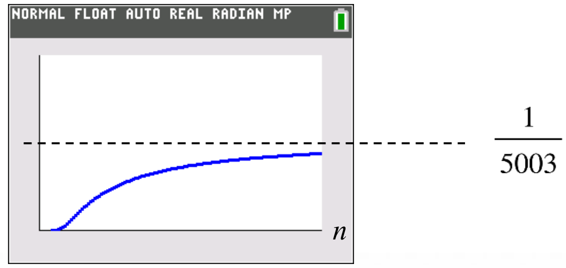


16. a. If $a_n = \left(\dfrac{n - \ln 5003}{n}\right)^n$,then use the cool fact that $\lim\limits_{n\to\infty}\left(1+\dfrac{r}{n}\right)^n = e^r$ where $r = -\ln 5003$.

Write $\dfrac{n-\ln 5003}{n} = \dfrac{n}{n} - \dfrac{\ln 5003}{n}$

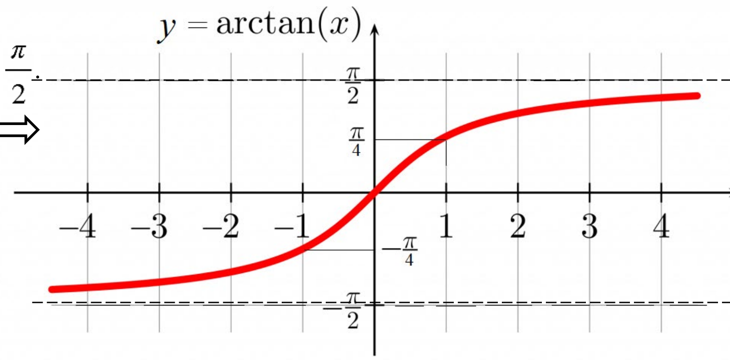
We have $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}\left(1+\dfrac{-\ln 5003}{n}\right)^n = e^{-\ln 5003} = e^{\ln 5003^{-1}} = 5003^{-1} = \boxed{\dfrac{1}{5003}}$.

A graph is not easy to produce (nor necessary), but a_n is monotonic and has a least upper bound of $\dfrac{1}{5300}$. Thus a_n **converges**.



b. If $a_n = \sqrt{5003}\tan^{-1}n$, then use the cool fact that $\lim\limits_{n\to\infty}\tan^{-1}n = \dfrac{\pi}{2}$.

It is helpful to know this graph of the inverse tangent function $\Rightarrow$

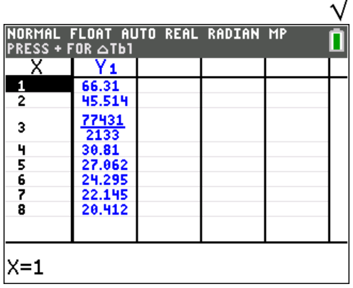


We have $\lim\limits_{n\to\infty}\sqrt{5003}\tan^{-1}n = \sqrt{5003}\lim\limits_{n\to\infty}\tan^{-1}n$

$= \boxed{\sqrt{5003}\cdot\dfrac{\pi}{2}}$.

For $n \geq 0$, the sequence a_n is monotonic and the limit is the least upper bound. Thus a_n **converges**.

c. Explore the table for $a_n = \dfrac{71-\sqrt{22x}}{\sqrt{x}} = \dfrac{71}{\sqrt{x}} - \dfrac{\sqrt{22x}}{\sqrt{x}} = \dfrac{71}{\sqrt{x}} - \sqrt{22}$ to see that the sequence decreases.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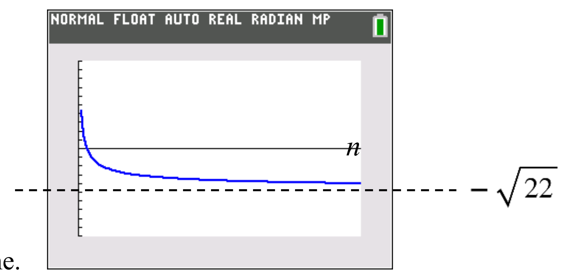
X	Y1
1	66.31
2	45.514
3	77431/2133
4	30.81
5	27.062
6	24.295
7	22.145
8	20.412

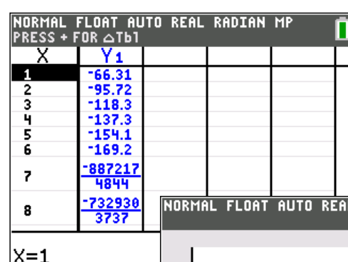
X=1

The graph shows that a_n is monotone.



Use the limit rules to find the limit: $\lim\limits_{n\to\infty}\left(\cancelto{0}{\dfrac{71}{\sqrt{x}}} - \sqrt{22}\right) = \boxed{-\sqrt{22}}$ which is the greatest lower bound. Thus a_n **converges**.

d. Explore the table for $a_n = \dfrac{\sqrt{22x}-71x}{\sqrt{x}}$ to see that the sequence decreases.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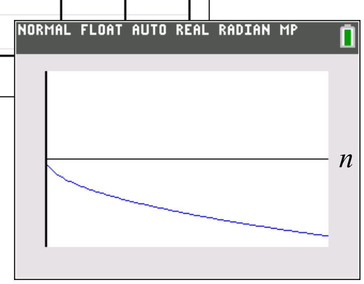
X	Y1
1	-66.31
2	-95.72
3	-118.3
4	-137.3
5	-154.1
6	-169.2
7	-887217/4844
8	-732930/3737

X=1

$a_n = \dfrac{\sqrt{22x}-71x}{\sqrt{x}} = \dfrac{\sqrt{22x}}{\sqrt{x}} - \dfrac{71x}{\sqrt{x}} = \sqrt{22} - \dfrac{71x}{x^{1/2}} = \sqrt{22} - 71x^{1/2} = \sqrt{22} - 71\sqrt{x}$

We have $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}\left(\sqrt{22} - 71\sqrt{x}\right) = \sqrt{22} - \infty = \boxed{-\infty}$.



The sequence a_n has no lower bound. Thus a_n **diverges**.