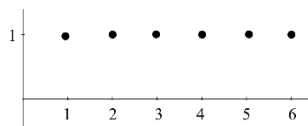


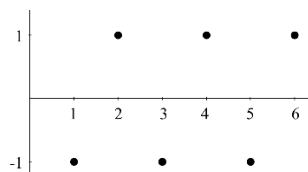
1. Consider the sequence $\{r^n\}$

Look at graphs of $y = r^n$ for $r = 1$, for $r = -1$, for $r = 0.5$, for $r = -0.5$, for $r = 2$, and for $r = -2$.

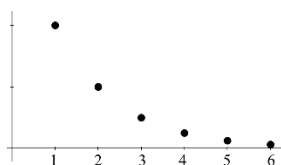
$y = (1)^n = 1$. We have $\lim_{n \rightarrow \infty} (1)^n = 1$.



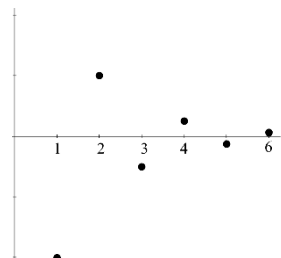
$y = (-1)^n$ oscillates between -1 and 1 . We have $\lim_{n \rightarrow \infty} (-1)^n$ does not exist.



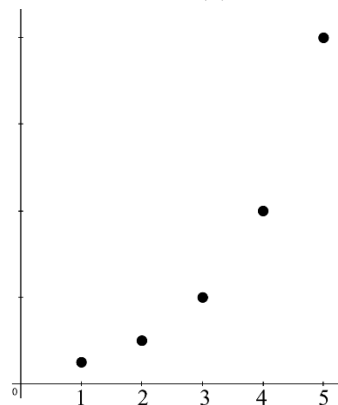
$y = (0.5)^n$ We have $\lim_{n \rightarrow \infty} (0.5)^n = 0$.



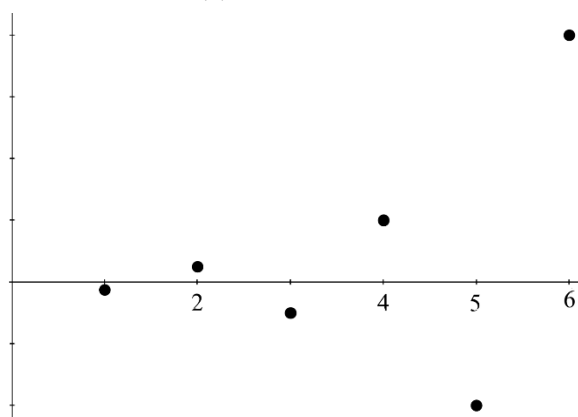
$y = (-0.5)^n$ We have $\lim_{n \rightarrow \infty} (-0.5)^n = 0$.



$y = (2)^n$ We have $\lim_{n \rightarrow \infty} (2)^n = \infty$.



$y = (-2)^n$ We have $\lim_{n \rightarrow \infty} (-2)^n$ does not exist.



- For what values of r does the sequence $\{r^n\}$ converge? $-1 < r \leq 1$
- For what values of r does the sequence $\{r^n\}$ oscillate? $r < 0$

2. Complete. The series $\sum_{k=0}^{\infty} a(r)^k = a + ar + ar^2 + \dots$ converges to the expression $\sum_{k=0}^{\infty} a(r)^k = a + ar + ar^2 + \dots = \frac{a}{1-r}$ under the following values of r : $|r| < 1$ or $-1 < r < 1$

Involves a and r .

Note: $\lim_{n \rightarrow \infty} r^n = 0$ for $|r| < 1$ so $\lim_{n \rightarrow \infty} \frac{a(1-r^n)}{1-r} = \frac{a(1-0)}{1-r} = \frac{a}{1-r}$

3. Complete: $\sum_{k=0}^{\infty} 400(1.10)^k = \infty$ Write in the box an exact number or DNE or ∞ or $-\infty$.

This is a geometric series with $r = 1.10 > 1$. It is the sum of the series in Question 9, climbing without bound to infinity.

4. If $\sum_{k=0}^{\infty} \frac{282}{13^{k-1}}$ were written as $\sum_{k=0}^{\infty} ar^k$, report a and r . $a = 3666$, $r = \frac{1}{13}$.

5. Complete: $\sum_{k=0}^{\infty} \frac{282}{13^{k-1}} = 3971.5$ Write in the box an exact number or DNE or ∞ or $-\infty$.

Write the series in expanded form as

$$\frac{282}{13^{-1}} + \frac{282}{13^0} + \frac{282}{13^1} + \dots = 282 \cdot 13 + 282 + \frac{282}{13} + \dots = \frac{3666}{1 - \frac{1}{13}} = \frac{3666}{\frac{12}{13}} = 3666 \cdot \frac{13}{12} = 3971.5$$

We can also write $\sum_{k=0}^{\infty} \frac{282}{13^{k-1}} = \sum_{k=0}^{\infty} \frac{282}{13^k 13^{-1}} = \sum_{k=0}^{\infty} \frac{282 \cdot 13}{13^k} = \sum_{k=0}^{\infty} 3666 \cdot \frac{1^k}{13^k} = \sum_{k=0}^{\infty} 3666 \cdot \left(\frac{1}{13}\right)^k$

6. The series $\sum_{k=0}^{\infty} ar^k$ converges to 5. If $a = 9.5$, what is the value of r ? Complete: $\sum_{k=0}^{\infty} 9.5 \left(\frac{\quad}{\quad}\right)^k = 5$

Show work.

$$5 = \frac{a}{1-r} = \frac{9.5}{1-r}$$

$$5(1-r) = 9.5$$

$$1-r = \frac{9.5}{5} = 1.9$$

$$1-r = 1.9$$

$$1-1.9 = r$$

$$-0.9 = r$$

7. The series $\sum_{k=0}^{\infty} ar^k$ converges to 5. If $r = \frac{1}{25}$, what is the value of a ? Complete: $\sum_{k=0}^{\infty} \left(\frac{\quad}{25}\right)^k = 5$

Show work.

$$5 = \frac{a}{1-r} = \frac{a}{1-\frac{1}{25}} = \frac{a}{\frac{24}{25}}$$

$$a = 5 \cdot \frac{24}{25} = \frac{24}{5}$$

8. Complete: $\frac{2 \cdot 124}{125} + \frac{2 \cdot 124^2}{125^2} + \frac{2 \cdot 124^3}{125^3} + \dots =$ Write in the box an exact number or DNE or ∞ or $-\infty$.

The first term, $\frac{2 \cdot 124}{125}$, is a .

The ratio $r = \frac{124}{125}$. So $\frac{a}{1-r} = \frac{\frac{2 \cdot 124}{125}}{1 - \frac{124}{125}} = \frac{\frac{2 \cdot 124}{125}}{\frac{1}{125}} = \frac{2 \cdot 124 \cdot \frac{1}{125}}{1 \cdot \frac{1}{125}} = 2 \cdot 124 = 248$

If we write it in sigma notation we can use our calculator to check:

One way to check:

NORMAL FLOAT AUTO REAL RADIAN HP

$$\sum_{k=0}^{2000} \left(\frac{2 \cdot 124}{125} \cdot \left(\frac{124}{125} \right)^k \right)$$

247.999974

Another way:

NORMAL FLOAT AUTO REAL RADIAN HP

$$\sum_{k=1}^{2000} \left(2 \cdot \left(\frac{124}{125} \right)^k \right)$$

247.9999738

9. Once per year Richie Rich deposits an amount of \$400 in an account which pays 10% interest per year, compounded annually, with additional deposits of \$400 continually made at the end of the year.

If B_n is the balance in the account, in dollars, immediately after Richie makes the n th deposit, then we can write $B_1 = \$400$.

- a. Complete the table to find the following. Report to the nearest \$0.01.

- the balance, B_2 , of the account on the day immediately after the second deposit.
- the balance, B_3 , of the account on the day immediately after the third deposit.
- the balance, B_4 , of the account on the day immediately after the fourth deposit.

n , # Deposits	B_n
1	\$400
2	840
3	1324
4	1856.40

- b. Suppose Richie makes 422 deposits. Which is true about the sum B_{422} ?

The balance, B_{422} , of the account on the day immediately after the 422nd deposit is exactly

- $B_{422} = 400 \cdot 10^{422} + 400 \cdot 10^{421} + \dots + 400 \cdot 10^2 + 400 \cdot 10 + 400$
- $B_{422} = 400 \cdot 1.10^{423} + 400 \cdot 1.10^{422} + \dots + 400 \cdot 1.10^2 + 400 \cdot 1.10 + 400$
- $B_{422} = 400 \cdot 10^{423} + 400 \cdot 10^{422} + \dots + 400 \cdot 10^2 + 400 \cdot 10 + 400$
- $B_{422} = 400 \cdot 1.10^{422} + 400 \cdot 1.10^{421} + \dots + 400 \cdot 1.10^2 + 400 \cdot 1.10 + 400$
- $B_{422} = 400 \cdot 1.10^{421} + 400 \cdot 1.10^{420} + \dots + 400 \cdot 1.10^2 + 400 \cdot 1.10 + 400$
- $B_{422} = 400 \cdot 10^{421} + 400 \cdot 10^{420} + \dots + 400 \cdot 10^2 + 400 \cdot 10 + 400$

NORMAL FLOAT AUTO REAL RADIAN HP

400

1.1Ans+400

1.1Ans+400

1.1Ans+400

1856.4

NORMAL FLOAT AUTO REAL RADIAN HP

Plot1 Plot2 Plot3

$Y_1 = \frac{400(1-1.10^X)}{1-1.10}$

- c. The balance, B_{422} , of the account on the day immediately after the 422nd deposit is approximately

- $B_{422} \approx \$1291712354137103000000$
- $B_{422} \approx \$1067530871187688000000$
- $B_{422} \approx \$1174283958306457000000$
- $B_{422} \approx \$1188774622351958700000$
- $B_{422} \approx \$14490664045501680000$
- The value of B_{422} can not be computed.

NORMAL FLOAT AUTO REAL RADIAN HP

Y1=

Y1=1.174283958307E21

calculate $\frac{400(1-1.10^{422})}{1-1.10}$

10. Complete the boxes and evaluate each of the following series. If it diverges to ∞ , then insert ∞ in the answer box.

a. $f(x) = \sum_{k=0}^{\infty} \frac{1380}{5^{2-k}} = 55.2 + 276 + 1380 + 6900 + \dots$

i. $a = 55.2$ $r = 5 = \frac{276}{55.2} = \frac{1380}{276} = \frac{6900}{1380} = 5$

ii. $f(x) = \sum_{k=0}^{\infty} \frac{1380}{5^{2-k}} = 55.2 + 276 + 1380 + 6900 + \dots = \infty$

iii. Give a reason for your claim in part ii. that does not have anything to do with technology.

geometric series with $r=5$ which is >1 ; terms increase to ∞

b. $f(x) = \sum_{k=0}^{\infty} \frac{1380}{5^{k-2}} = 34500 + 6900 + 1380 + 276 + 55.2 + \dots$

i. $a = 34500$ $r = 0.2$ or $\frac{1}{5}$

ii. $f(x) = \sum_{k=0}^{\infty} \frac{1380}{5^{k-2}} = 34500 + 6900 + 1380 + 276 + 55.2 + \dots = 43125$

iii. Give a reason for your claim in part ii. that does not have anything to do with technology.

$\frac{a}{1-r} = \frac{34500}{1-0.2} = \frac{34500}{0.8} = 43125$ since $r = \frac{1}{5}$ which is <1

11. Professor Snape needs to create a potion for Remus Lupin to address the negative effects of his lycanthropy. Unfortunately, this medication takes a very long time to stabilize. Snape wants the stabilization level to eventually be 800 mg. For this to happen, Lupin must take the potion once per day in perpetuity. Lupin's body will eliminate only 3% of the medication between each dose. Answer the questions below. $r = 0.97$

a. What dosage should Professor Snape prescribe so that the drug stabilization level will be 800 mg?

Lupin must take **24** mg each day. Show your calculations. $\frac{a}{1-r} = 800 \Rightarrow \frac{a}{1-0.97} = 800 \Rightarrow a = .03 \cdot 800 =$

b. Create a formula which gives the amount of medication that is present, in mg, in Lupin's body right after the x th dose of the amount prescribed in part a. $A(x) = 800 - 800(0.97)^x$

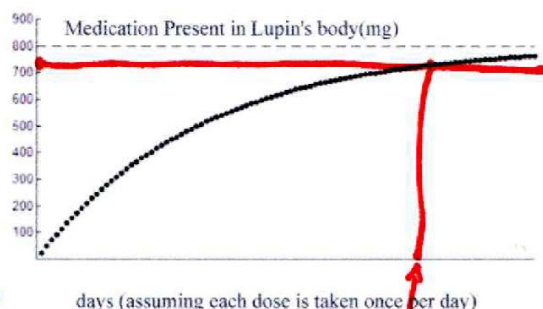
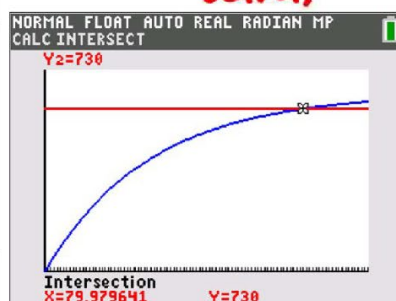
c. To the right is a graph of the formula in part b.

The drug will take effect when the medication level in Lupin's body is first within 730 mg. How many days will it take for the drug to take effect? It will take **80** days to reach a level of 730 mg, assuming Lupin takes one dose every day as prescribed. No work need be shown. Utilize your technology.

Solve with a table, graphical intersection, or a numeric solver.

NORMAL FLOAT AUTO REAL RADIAN MP				
PRESS $\blacktriangle$ TO EDIT FUNCTION				
X	Y1			
78	725.65			
79	727.88			
80	730.04			
81	732.14			
82	734.18			
83	736.15			
84	738.07			
85	739.93			
86	741.73			
87	743.48			
88	745.17			

$Y1 = 730.04339446493$



NORMAL FLOAT AUTO REAL RADIAN MP			
Plot1	Plot2	Plot3	
$Y1 = 800 - 800(.97)^x$			
$Y2 = 730$			

12. Find the exact value of k for which $e^k + e^{2k} + e^{3k} + e^{4k} + \dots = 99$

$$e^k + e^{2k} + e^{3k} + e^{4k} + \dots = e^k + (e^k)^2 + (e^k)^3 + (e^k)^4 + \dots$$

$$a = e^k, r = e^k \text{ so } \frac{a}{1-r} = \frac{e^k}{1-e^k} = 99$$

$$e^k = 99(1-e^k)$$

$$e^k = 99 - 99e^k$$

$$e^k + 99e^k = 99$$

$$100e^k = 99$$

$$e^k = 0.99$$

$$\ln e^k = \ln 0.99$$

$$k = \ln 0.99$$

Check: If $k = \ln 0.99$, then

$$e^k + e^{2k} + e^{3k} + e^{4k} + \dots = e^{\ln 0.99} + (e^{\ln 0.99})^2 + (e^{\ln 0.99})^3 + (e^{\ln 0.99})^4 + \dots$$

$$= 0.99 + (0.99)^2 + (0.99)^3 + (0.99)^4 + \dots$$

$$\text{So } a = 0.99 \text{ and } r = 0.99 \text{ and } 0.99 + (0.99)^2 + (0.99)^3 + (0.99)^4 + \dots = \frac{a}{1-r} = \frac{0.99}{1-0.99} = \frac{0.99}{0.01} = 99$$

13. Write in the box an **exact** number or DNE or ∞ or $-\infty$. Then report if the sequence a_n converges or diverges.

Hint: Remember $\lim_{n \rightarrow \infty} (1 + \frac{r}{n})^n = e^r$.

a. If $a_n = \left(\frac{n - \ln 5003}{n} \right)^n$, then $\lim_{n \rightarrow \infty} a_n = \boxed{\frac{1}{5003}}$. The sequence converges.

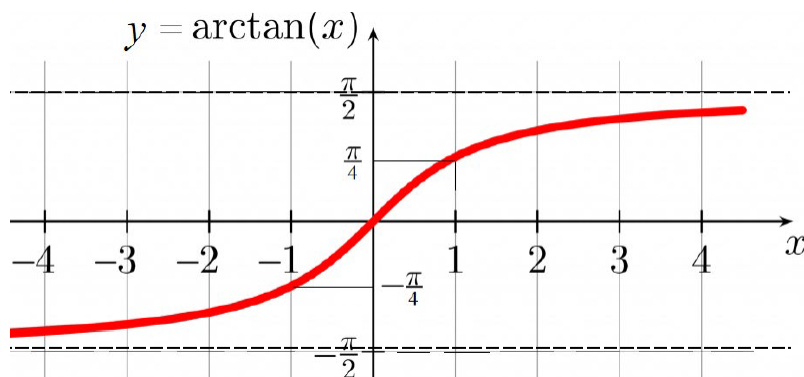
We use the fun fact that $\lim_{n \rightarrow \infty} (1 + \frac{r}{n})^n = e^r$.

$$\text{If } a_n = \left(\frac{n - \ln 5003}{n} \right)^n = \left(1 - \frac{\ln 5003}{n} \right)^n = \left(1 + \frac{-\ln 5003}{n} \right)^n, \text{ then } \lim_{n \rightarrow \infty} a_n = e^{-\ln 5003} = \frac{1}{e^{\ln 5003}} = \frac{1}{5003}.$$

b. If $a_n = \sqrt{5003} \tan^{-1} n$, then $\lim_{n \rightarrow \infty} a_n = \boxed{\frac{\pi\sqrt{5003}}{2}}$. The sequence converges.

$$\text{If } a_n = \sqrt{5003} \tan^{-1} n, \text{ then use the cool fact that } \lim_{n \rightarrow \infty} \tan^{-1} n = \frac{\pi}{2} \text{ so } \lim_{n \rightarrow \infty} \sqrt{5003} \tan^{-1} n = \frac{\pi\sqrt{5003}}{2}.$$

It is helpful to know this graph of the inverse tangent function.



c. If $a_n = \frac{70 - \sqrt{22n}}{\sqrt{n}}$, then $\lim_{n \rightarrow \infty} a_n = \boxed{-\sqrt{22}}$. The sequence {converges, diverges}.

We have $a_n = \frac{70 - \sqrt{22n}}{\sqrt{n}} = \frac{70}{\sqrt{n}} - \frac{\sqrt{22n}}{\sqrt{n}} = \frac{70}{\sqrt{n}} - \sqrt{22}$ and $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{70}{\sqrt{n}} - \sqrt{22} = -\sqrt{22}$

d. If $a_n = \frac{\sqrt{22n} - 70n}{\sqrt{n}}$, then $\lim_{n \rightarrow \infty} a_n = \boxed{-\infty}$. The sequence {converges, diverges}.

We have $a_n = \frac{\sqrt{22n} - 70n}{\sqrt{n}} = \frac{\sqrt{22n}}{\sqrt{n}} - \frac{70n}{\sqrt{n}} = \frac{\sqrt{22}\sqrt{n}}{\sqrt{n}} - \frac{70n}{\sqrt{n}} = \sqrt{22} - \frac{70n\sqrt{n}}{n} = \sqrt{22} - 70\sqrt{n}$
and $\lim_{n \rightarrow \infty} a_n = -\infty$

e. If $a_n = \left(1 - \frac{71}{n}\right)^n$, then $\lim_{n \rightarrow \infty} a_n = \boxed{e^{-71}}$. The sequence {converges, diverges}.

We use the fun fact that $\lim_{n \rightarrow \infty} \left(1 + \frac{r}{n}\right)^n = e^r$.

If $a_n = \left(1 - \frac{71}{n}\right)^n = \left(1 + \frac{-71}{n}\right)^n$, then $\lim_{n \rightarrow \infty} a_n = e^{-71}$.

14. Consider the sequence given by the recurrence relation $a_{n+1} = 0.95a_n + 8.2$, $a_1 = 8.2$

a. The sequence converges to a limit L . Give the exact value of L . $L = \boxed{164}$

Since 0.95 is so close to 1, convergence is slow; the graph takes a very long time before it gets close to its horizontal asymptote.

b. Convergence occurs when $a_{n+1} = a_n$. Use this fact to rewrite the above recurrence relation into an equation that involves L .

Equation: $L = 0.95L + 8.2$

$L - 0.95L = 8.2$
 $L(1 - 0.95) = 8.2$

c. Solve the equation in part b to justify your claim in part a.

$L = \frac{8.2}{1 - 0.95} = 164$

d. Complete the boxes below to write the next two terms of the series in long form. Each subsequent term involves a numerical expression containing 0.95 and 8.2.

$8.2 + \boxed{8.2(0.95)} + \boxed{8.2(0.95)^2} + \dots$

e. Without using sigma notation, write an expression that gives the n th partial sum of this series $S_n =$ $\boxed{8.2(1 - .95^n)}$
i.e., the sum of the series of n terms.

$1 - 0.95$

f. Enter your expression from part e in your grapher and scroll a table to find the value of n for which the sum first surpasses 150.

The number of terms $n =$

$\boxed{48}$

X	Y1
1	8.2
2	15.99
3	23.391
4	30.421
47	149.28
48	150.02
49	150.72

Y1 = $8.2(1 - 0.95^X)$
TABLE SETUP
TblStart=1000
TblEnd=500
Indnt: 0.000 Ask
Depnd: 0.000 Ask

X	Y1
1000	164
1500	164
2000	164
2500	164
3000	164
3500	164
4000	164
4500	164
5000	164
5500	164

Check sum is 164: