1a. $r = \csc\theta$

$r = \frac{1}{\sin\theta}$

$r\sin\theta = 1$

$\boxed{y = 1}$

Check w/ grapher.

1b. $r = \frac{\tan\theta}{\cos\theta - \sin\theta}$

$r(\cos\theta - \sin\theta) = \tan\theta$

$r\cos\theta - r\sin\theta = \tan\theta$

$x - y = \frac{y}{x}$

Multiply all terms by x

$x^2 - xy = y$

Get all y terms on one side

$x^2 = y + xy$

Factor out y

$x^2 = y(1+x)$

Divide $\boxed{y = \frac{x^2}{1+x}}$

1c. $r = \frac{1}{\cos\theta + \sin\theta}$

$r(\cos\theta + \sin\theta) = 1$

$r\cos\theta + r\sin\theta = 1$

$x + y = 1$

$\boxed{y = 1 - x}$

Check w/ grapher

1d. $r = \frac{2\csc\theta}{\cot\theta + r\cos\theta}$

$r = 2\csc\theta \cdot \frac{1}{\cot\theta + r\cos\theta}$

$r = 2 \cdot \frac{1}{\sin\theta} \cdot \frac{1}{(\cot\theta + r\cos\theta)}$

So

$r\sin\theta(\cot\theta + r\cos\theta) = 2$

$y \cdot \left(\frac{x}{y} + x\right) = 2$

Distribute:

$y \cdot \frac{x}{y} + xy = 2$

$x + xy = 2$

$xy = 2 - x$

$y = \boxed{\frac{2-x}{x}}$ or $\boxed{\frac{2}{x} - 1}$

1e. $r^2\cos\theta + r\tan\theta = \sec\theta$

Multiply all terms by $\frac{1}{r}$

$r\cos\theta + \tan\theta = \frac{1}{r\cos\theta}$

$x + \frac{y}{x} = \frac{1}{x}$

$\frac{y}{x} = \frac{1}{x} - x$

Multiply all terms by x

check w/ grapher $\boxed{y = 1 - x^2}$

1f. $r^2 = \sec^2\theta\tan\theta$

$r^2 = \frac{1}{\cos^2\theta} \frac{\sin\theta}{\cos\theta}$

$r^2\cos^2\theta \cdot \cos\theta = \sin\theta$

Multiply both sides by r

$r^2\cos^2\theta \cdot r\cos\theta = r\sin\theta$

$x^2 \cdot x = y$

$\boxed{y = x^3}$

check w/ grapher.

2. Given the Cartesian equation in terms of x and y, write the polar equation in terms of r and θ. Your equation should begin with "$r =$"

a. $x^2 + y^2 = x + y$

$r^2 = r\cos\theta + r\sin\theta$

$\boxed{r = \cos\theta + \sin\theta}$

b. $x^2\left(x^2 + y^2\right) = y^2$

$\left(x^2 + y^2\right) = \dfrac{y^2}{x^2}$

$r^2 = \tan^2\theta$ Take square roots of both sides.

$\boxed{r = \tan\theta}$

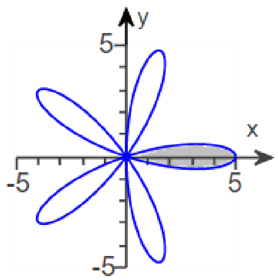
c. $y = 3 - 2x$

$r\sin\theta = 3 - 2r\cos\theta$

$r\sin\theta + 2r\cos\theta = 3$

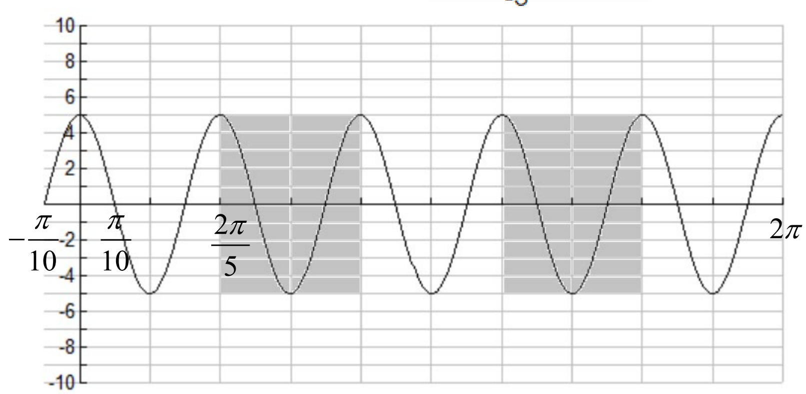
$r(\sin\theta + 2r\cos\theta) = 3$

$\boxed{r = \dfrac{3}{2\cos\theta + \sin\theta}}$



3. Recall the area from $\theta = \alpha$ to $\theta = \beta$ inside a polar graph is $\int_\alpha^\beta \frac{1}{2} r^2 \, d\theta$

a. Find the exact area of the region inside one leaf of the 5-leaved rose $r = 5\cos 5\theta$
You can use the FNINT command, but provide an exact area.



Solve when $r = 5\cos 5\theta = 0$.
The smallest negative value is the solution to

$$5\theta = -\frac{\pi}{2}.$$

Divide both sides by 5: $\theta = -\dfrac{\pi}{10}$

The smallest positive value is the solution to

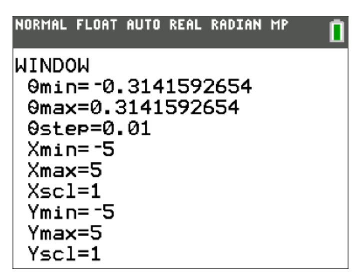
$$5\theta = \frac{\pi}{2}.$$

Divide both sides by 5: $\theta = \dfrac{\pi}{10}$

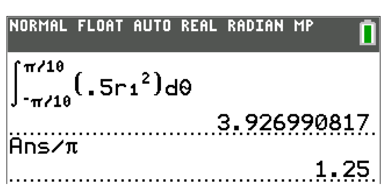
We can also solve this graphically.

Sketch a cosine graph $y = 5\cos 5\theta$. There are 5 cycles in one interval of $[0, 2\pi]$, so a first cycle happens on $[0, \frac{2\pi}{5}]$.

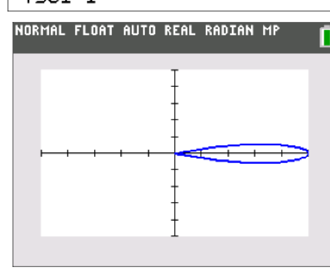
We can sketch one petal in the polar grapher if we set θmin $= -\dfrac{\pi}{10}$ and θ max $= \dfrac{\pi}{10}$.



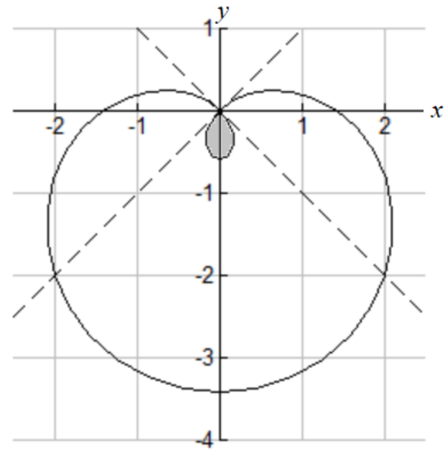
r1=5cos(5θ)



$$\int_{\boxed{-\pi/10}}^{\boxed{\pi/10}} \boxed{\frac{1}{2}(5\cos 5\theta)^2} \, d\theta = \boxed{\frac{5}{4}\pi} \text{ or } \boxed{1.25\pi}$$



b. Set up the integral to calculate the area of the region inside the inner loop of the limaçon $r = \sqrt{2} - 2\sin\theta$. Use the FNINT command to find the area and approximate it the area to two decimal places.



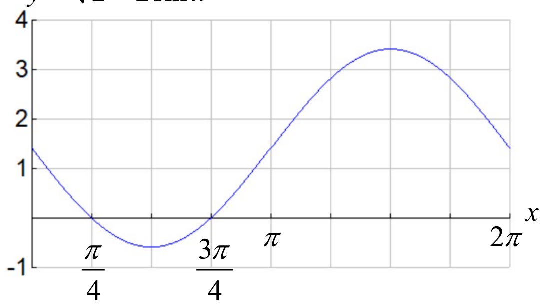
To find the integration limits, find where $r = \sqrt{2} - 2\sin\theta = 0$ where $0 \le \theta < 2\pi$, since this will be where the inner loop starts and ends.
TIP: The dashed lines in the above graph are the polar equations $\theta = \alpha$ and $\theta = \beta$, where α and β are the lower and upper limits of integration. You can enter these values in your polar grapher as *θmin* and *θmax* to check that you have sketched only the inner loop.
Solve $r = \sqrt{2} - 2\sin\theta = 0$ graphically or algebraically.

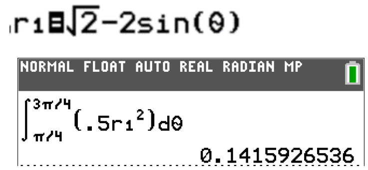
$$2\sin\theta = \sqrt{2}$$

$$\sin\theta = \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

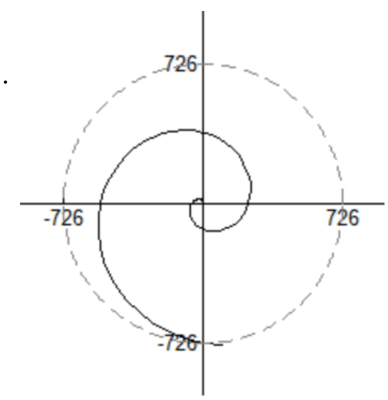
$y = \sqrt{2} - 2\sin x$





$$\int_{\boxed{\pi/4}}^{\boxed{3\pi/4}} \boxed{\frac{1}{2}(\sqrt{2} - 2\sin\theta)^2}\, d\theta \approx \boxed{0.14}$$

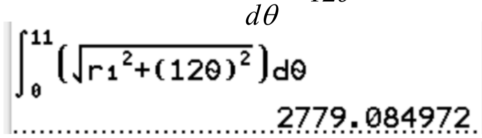
3. The arc length from $\theta = 0$ to $\theta = 11$ of a polar spiral $r = 6\theta^2$ is given by $\int_0^{11} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$.



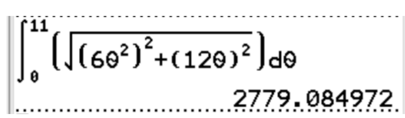
Report the arc length correct to the nearest whole number.
You can use the FNINT command. Round to the nearest whole number.

$$r = 6\theta^2$$

$$\frac{dr}{d\theta} = 12\theta$$



or



$$\int_0^{11} \sqrt{\boxed{(6\theta^2)^2 + (12\theta)^2}}\, d\theta \approx \boxed{2779}$$

4. Find the indefinite integrals. Show work.

a. $\int \tan^9 x \sec^2 x \, dx =$ $\frac{1}{10}\tan^{10} x$ $+ C$

$u = \tan x$
$du = \sec^2 x dx$ $\Rightarrow \int \tan^9 x \sec^2 x dx = \int u^9 du = \frac{1}{10} u^{10} + C$

$= \boxed{\frac{1}{10}\tan^{10} x + C}$

b. $\int \cos^2 \theta \, d\theta =$ $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$ $+ C$

Use the Pythagorean Double Identity $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$\int \cos^2\theta d\theta = \frac{1}{2}\int (1 + \cos 2\theta) d\theta = \frac{1}{2}\int d\theta + \frac{1}{2}\cdot\frac{1}{2}\int \underbrace{\cos 2\theta}_{\cos u} \cdot \underbrace{2 d\theta}_{du}$

$= \boxed{\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C}$

c. $\int \sin^3 x \cos^6 x \, dx =$ $\frac{1}{9}\cos^9 x - \frac{1}{7}\cos^7 x$ $+ C$

If you let $u = \sin x$, then we need one $\cos x$ for du, which leaves $\int u^3 \cos^5 x dx$ and Pythagoras can't help us unless it is even.

So let $u = \cos x$
$du = -\sin x dx$ $\Rightarrow \int \cos^6 x \cdot \sin^2 x \cdot \underbrace{\sin x dx (-1)}_{du}(-1)$

$= \int u^6 \cdot \sin^2 x \; du \cdot (-1)$

Pythagoras to the rescue! $\sin^2 x = 1 - \cos^2 x = 1 - u^2$

$= \int u^6 \cdot (-1)(1 - \cos^2 x) du$

$= \int u^6 (-1)(1 - u^2) du = \int u^6 (u^2 - 1) du$

$= \int (u^8 - u^6) du$

$= \frac{1}{9}u^9 - \frac{1}{7}u^7 + C = \boxed{\frac{1}{9}\cos^9 x - \frac{1}{7}\cos^7 x + C}$

5. Consider the integral $\int \frac{\sin\theta}{\cos^2\theta} d\theta$.

a. Select which of these is the antiderivative for the integral $\int \frac{\sin\theta}{\cos^2\theta} d\theta$.

A. $\sin\theta + C$	B. $\cos\theta + C$	C. $\tan\theta + C$	D. $\csc\theta + C$	E. $\sec\theta + C$	F. $\cot\theta + C$
G. $-\sin\theta + C$	H. $-\cos\theta + C$	I. $-\tan\theta + C$	J. $-\csc\theta + C$	K. $-\sec\theta + C$	L. $-\cot\theta + C$
M. All of these.	N. None of these.				

b. Explain your reasoning for your selection.

Method 1: $\int \frac{\sin\theta}{\cos^2\theta} d\theta = \int \frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta} d\theta = \int \sec\theta \tan\theta d\theta = \boxed{\sec\theta + C}$ from formula sheet

Method 2: Let $u = \cos\theta$
$du = -\sin\theta d\theta$

$\int \frac{\sin\theta}{\cos^2\theta} d\theta = -\int \frac{1}{\cos^2\theta}(-\sin\theta) d\theta = -\int u^{-2} du = \frac{1}{u} + C$

$= \frac{1}{\cos\theta} + C$

$= \boxed{\sec\theta + C}$

6. Consider $\int \sec^{14} x \tan^{17} x \, dx$

a. Suppose we let $u = \tan x$. Then $du =$ $\sec^2 x$ dx

Then we can write $\int \sec^{14} x \tan^{17} x \, dx = \int \boxed{u^{17}(1+u^2)^6} \, du.$

Your answer is a binomial in terms of u raised to a power multiplied by u raised to a power.
Do not multiply it out. Do not find the antiderivative. Just leave it as a polynomial.

$$\int \sec^{14} x \tan^{17} x \, dx = \int u^{17} \sec^{12} x \cdot \underbrace{\sec^2 x \, dx}_{du}$$
$$= \int u^{17} (\sec^2 x)^6 \, du$$
$$= \int u^{17} (1+\tan^2 x)^6 \, du = \int u^{17}(1+u^2)^6 \, du$$

use $1+\tan^2 x = \sec^2 x$

b. Suppose we let $w = \sec x$. Then $dw =$ $\sec w \tan w$ dx

Then we can write $\int \sec^{14} x \tan^{17} x \, dx = \int \boxed{w^{13}(w^2-1)^8} \, dw.$

Your answer is a binomial in terms of w raised to a power multiplied by w raised to a power.
Do not multiply it out. Do not find the antiderivative. Just leave it as a polynomial.

$$\int \sec^{14} x \tan^{17} x \, dx = \int \sec^{13} x \tan^{16} x \, \underbrace{\sec x \tan x \, dx}_{du}$$
$$= \int w^{13} (\tan^2 x)^8 \, du = \int w^{13}(w^2-1)^8 \, dw$$

use $\sec^2 x - 1 = \tan^2 x$