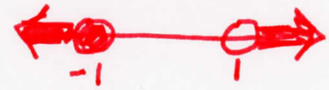
Practice Questions from Section 10.1 – 10.3

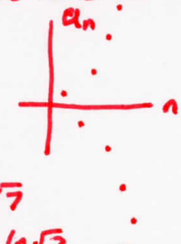
1. **a.** For what values of r does the sequence $\{r^n\}$ converge? $-1 < r \leq 1$

b. For what values of r does the sequence $\{r^n\}$ diverge? $r \leq -1,\ r > 1$



2. Complete: $\lim\limits_{n\to\infty} (-1)^{n+1}(2)^n = \boxed{\text{DNE}}$ Write in the box an exact number or DNE or ∞ or $-\infty$.

This diverges because it oscillates to ∞ and to $-\infty$



3. Complete: $\lim\limits_{n\to\infty}\left(1+\dfrac{\ln\sqrt{7}}{n}\right)^n = \boxed{\sqrt{7}}$ Write in the box an exact number or DNE or ∞ or $-\infty$.

This is a case of $\lim\limits_{n\to\infty}\left(1+\frac{r}{n}\right)^n = e^r$ where $r = \ln\sqrt{7}$ so $e^{\ln\sqrt{7}} = \sqrt{7}$.

4. Consider the sequence given by $\{a_n\}_{n=1}^{\infty}$: $\dfrac{7}{32}, \dfrac{7^2}{34}, \dfrac{7^3}{36}, \dfrac{7^4}{38}, \dfrac{7^5}{40}, \ldots$

Find a formula for the nth term of the sequence in terms of n. $\{a_n\}_{n=1}^{\infty} = \boxed{\dfrac{7^n}{30+2n}}$

5. **a.** Write the next two terms of the sequence $\{a_n\}_{n=1}^{\infty}$: $\pi,\ -\pi,\ \pi,\ -\pi,$ π, $-\pi$

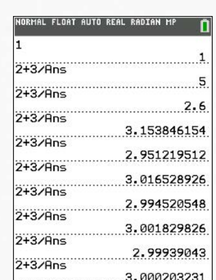
b. Find a formula for the nth term of the sequence in terms of n. $\{a_n\}_{n=1}^{\infty} = \boxed{(-1)^{n+1}\cdot\pi}$

↳ other formulas are possible such as $a_n = -\pi(-1)^n$

c. Write a recurrence relation that generates the above sequence.

$a_{n+1} = \boxed{-a_n}$, $a_1 = \boxed{\pi}$, for $n = 1, 2, 3, \ldots$.

6. Consider the sequence given by the recurrence relation $a_{n+1} = 2 + \dfrac{3}{a_n}$, $a_1 = 1$



a. The sequence converges to a limit L. Give the exact value of L. $L = \boxed{3}$

b. Convergence occurs when $a_{n+1} = a_n$. Use this fact to write an equation that involves L.

Equation: $2 + \frac{3}{L} = L$

c. Solve the equation in part **b** to justify your claim in part **a**. You will get two solutions. Reject the negative value of L.

Multiply both sides by $L \Rightarrow 2L + 3 = L^2$

$0 = L^2 - 2L - 3$

$0 = (L-3)(L+1)$

$L = 3,\ L = -1$ reject

7. Complete: $\sum\limits_{k=0}^{\infty} 400(1.10)^k = \boxed{\infty}$ Write in the box an exact number or DNE or ∞ or $-\infty$.

Geometric series with $r > 1$

8. Complete: $\sum\limits_{k=0}^{\infty} \dfrac{282}{13^{k-1}} = \boxed{3971.5}$ Write in the box an exact number or DNE or ∞ or $-\infty$.

If $\sum\limits_{k=0}^{\infty} \dfrac{282}{13^{k-1}}$ were written as $\sum\limits_{k=0}^{\infty} ar^k$, report a and r. $a =$ 3666, $r =$ 1/13.

$$= \sum_{k=0}^{\infty} \frac{282}{13^k \cdot 13^{-1}} = \sum_{k=0}^{\infty} \frac{282\cdot 13}{13^k} = \sum_{k=0}^{\infty} 3666\cdot\frac{1^k}{13^k}$$

$$\frac{a}{1-r} = \frac{3666}{1-1/13} = \frac{3666}{12/13} = \frac{3666\cdot 13}{12} = \boxed{3971.5}$$

9. The series $\sum_{k=0}^{\infty} ar^k$ converges to 5. If $a = 9.5$, what is the value of r? Complete: $\sum_{k=0}^{\infty} 9.5\left(\boxed{\frac{-9}{10}}\right)^k = 5$

Show work. $5 = \frac{a}{1-r} = \frac{9.5}{1-r} \Rightarrow 1-r = \frac{9.5}{5}$

$1 - r = 1.9 \Rightarrow r = 1 - 1.9 = -0.9 = \frac{-9}{10}$

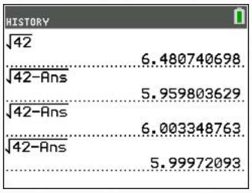
10. The series $\sum_{k=0}^{\infty} ar^k$ converges to 5. If $r = \frac{1}{25}$, what is the value of a? Complete: $\sum_{k=0}^{\infty} \boxed{\frac{24}{5}}\left(\frac{1}{25}\right)^k = 5$

Show work. $5 = \frac{a}{1-r} = \frac{a}{1 - 1/25} = \frac{a}{\frac{24}{25}}$ so $a = 5 \cdot \frac{24}{25} = \frac{24}{5}$

11. Consider the sequence $\sqrt{42}$, $\sqrt{42-\sqrt{42}}$, $\sqrt{42-\sqrt{42-\sqrt{42}}}$, $\sqrt{42-\sqrt{42-\sqrt{42-\sqrt{42}}}}$, ….

a. Write a recurrence relation that generates the above sequence.

$a_{n+1} = \boxed{\sqrt{42 - a_n}}$, $a_1 = \boxed{\sqrt{42}}$, for $n = 1, 2, 3, \ldots$



b. The sequence converges to a limit L. Give the exact value of L. $L = \boxed{6}$

c. Convergence occurs when $a_{n+1} = a_n$. Use this fact to rewrite the above recurrence relation into an equation that involves L.

Equation: $\sqrt{42 - L} = L$ $\Rightarrow 42 - L = L^2$

$0 = L^2 + L - 42$

d. ***Solve*** the equation part **c** to justify your claim in part **b**.

$0 = (L-6)(L+7) = 0$

$L = 6 \;||\; L = -7$

reject since $L > 0$

12. For what values of r does the series $\sum_{k=0}^{\infty} a(r)^k$ converge? $-1 < r < 1$

or $|r| < 1$

13. Consider the function $f(x) = \sum_{k=0}^{\infty} 9\left(\frac{x-4}{2}\right)^k$

a. Evaluate $f(3)$. Show work. $f(3) = \boxed{6}$ Write in the box an exact number or DNE or ∞ or $-\infty$.

$f(3) = \sum_{k=0}^{\infty} 9\left(-\frac{1}{2}\right)^k = \frac{a}{1-r}$ with $a = 9$, $r = -1/2$ $\Rightarrow \frac{9}{1-(-1/2)} = \frac{9}{\frac{3}{2}} = 9 \cdot \frac{2}{3} = 6$

b. For what values of x does $f(x)$ converge? Show work.

Solve $-1 < \frac{x-4}{2} < 1$ for x

Multiply all parts by 2: $-2 < x - 4 < 2$

Add 4 to all parts: $2 < x < 6$

$\boxed{2} < x < \boxed{6}$

14. Complete: $\frac{2 \cdot 124}{125} + \frac{2 \cdot 124^2}{125^2} + \frac{2 \cdot 124^3}{125^3} + \ldots = \boxed{250}$ Write in the box an exact number or DNE or ∞ or $-\infty$.

$= \sum_{k=1}^{\infty} 2\left(\frac{124}{125}\right)^k = \frac{a}{1-r}$ with $a = 2$, $r = \frac{124}{125}$ $\Rightarrow \frac{2}{1 - \frac{124}{125}} = \frac{2}{\frac{1}{125}} = \boxed{250}$

15. Consider the sequence given by the recurrence relation $a_{n+1} = 0.95a_n + 8.2, \ a_1 = 8.2$

a. The sequence converges to a limit L. Give the exact value of L. $L = \boxed{164}$

Since 0.95 is so close to 1, convergence is slow; the graph takes a very long time before it gets close to its horizontal asymptote.

b. Convergence occurs when $a_{n+1} = a_n$. Use this fact to rewrite the above recurrence relation into an equation that involves L.

Equation: $L = 0.95L + 8.2$

c. ***Solve*** the equation in part **b** to justify your claim in part **a**.

$L - 0.95L = 8.2$

$L(1-0.95) = 8.2$

$L = \dfrac{8.2}{1-0.95} = 164$

d. Complete the boxes below to write the next two terms of the series in long form.
Each subsequent term involves a numerical expression containing 0.95 and 8.2.

$8.2 + \boxed{8.2(0.95)} + \boxed{8.2(0.95)^2} + \ldots$

e. Without using sigma notation, write an expression that gives the nth partial sum of this series $S_n =$ i.e., the sum of the series of n terms.

$$S_n = \frac{\boxed{8.2}\left(1 - \boxed{.95}^{\boxed{n}}\right)}{\boxed{1-0.95}}$$

f. Enter your expression from part **e** in your grapher and scroll a table to find the value of n for which the sum first surpasses 150.

X	Y1
1	8.2
2	15.99
3	23.391
⋮	⋮
47	149.28
48	150.02
49	150.72

The number of terms $n = \boxed{48}$

Check sum is 164:

Y1 = $\dfrac{8.2(1-0.95^X)}{1-0.95}$

TABLE SETUP
TblStart=1000
ΔTbl=500
Indpnt: Auto Ask
Depend: Auto Ask

X	Y1
1000	164
1500	164
2000	164
2500	164
3000	164
3500	164
4000	164
4500	164
5000	164
5500	164
6000	164

16. Once per year Richie Rich deposits an amount of \$400 in an account which pays 10% interest per year, compounded annually, with **additional deposits of \$400 continually made at the end of the year**.

If B_n is the balance in the account, in dollars, immediately after Richie makes the nth deposit, then we can write $B_1 = \$400$.

a. Complete the table to find the following. Report to the nearest \$0.01.

i) the balance, B_2, of the account on the day immediately after the second deposit.
ii) the balance, B_3, of the account on the day immediately after the third deposit.
iii) the balance, B_4, of the account on the day immediately after the fourth deposit.

n, # Deposits	B_n
1	\$400
2	840
3	1324
4	1856.40

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400
400
1.1Ans+400
840
1.1Ans+400
1324
1.1Ans+400
1856.4

b. Suppose Richie makes 422 deposits. Which is true about the sum B_{422}?
The balance, B_{422}, of the account on the day immediately after the 422nd deposit is exactly

A. $B_{422} = 400\cdot10^{422} + 400\cdot10^{421} + \ldots + 400\cdot10^2 + 400\cdot10 + 400$

B. $B_{422} = 400\cdot1.10^{423} + 400\cdot1.10^{422} + \ldots + 400\cdot1.10^2 + 400\cdot1.10 + 400$

C. $B_{422} = 400\cdot10^{423} + 400\cdot10^{422} + \ldots + 400\cdot10^2 + 400\cdot10 + 400$

D. $B_{422} = 400\cdot1.10^{422} + 400\cdot1.10^{421} + \ldots + 400\cdot1.10^2 + 400\cdot1.10 + 400$

E. $B_{422} = 400\cdot1.10^{421} + 400\cdot1.10^{420} + \ldots + 400\cdot1.10^2 + 400\cdot1.10 + 400$ ← we have 422 deposits of 400

F. $B_{422} = 400\cdot10^{421} + 400\cdot10^{420} + \ldots + 400\cdot10^2 + 400\cdot10 + 400$

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Plot1 Plot2 Plot3
Y1 = $\dfrac{400(1-1.10^X)}{1-1.10}$

c. The balance, B_{422}, of the account on the day immediately after the 422nd deposit is approximately

A. $B_{422} \approx \$1291712354137103000000$

B. $B_{422} \approx \$1067530871187688000000$

C. $B_{422} \approx \$1174283958306457000000$ ←

D. $B_{422} \approx \$1188774622351958700000$

E. $B_{422} \approx \$1449066404550168000$

F. The value of B_{422} can not be computed.

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X	Y1
1	400
2	840
3	1324
4	1856.4
⋮	⋮
422	1.2E21
423	1.3E21
424	1.4E21
425	1.6E21
426	1.7E21
427	1.9E21
428	2.1E21
429	2.3E21
430	2.5E21

Y1=1.174283958307E21

calculate $\dfrac{400(1-1.10^{422})}{1-1.10}$