MA 16600
QUIZ 11 (HW 29-30) (25 pts)

Name **KEY**
Group ______

1. Integrate by parts. Show work. $\int xe^{5x}\,dx = \boxed{\tfrac{1}{5}xe^{5x} - \tfrac{1}{25}e^{5x}} + C$

$u =$ x $\quad dv =$ e^{5x} dx

$du =$ 1 $dx \quad v = \int e^{5x}\,dx = \frac{1}{5}\int e^{5x}5\,dx = \frac{1}{5}e^{5x}$

$$\int xe^{5x}\,dx = \tfrac{1}{5}xe^{5x} - \tfrac{1}{5}\int e^{5x}dx = \tfrac{1}{5}xe^{5x} - \tfrac{1}{5}\cdot\tfrac{1}{5}\int e^{5x}5dx = \tfrac{1}{5}xe^{5x} - \tfrac{1}{25}e^{5x} + C$$

2. Consider $\int \sec^{12}x\tan^{17}x\,dx$ Suppose we let $u = \sec x$. Then $du =$ $\sec x\tan x$ dx

Then we can write $\int \sec^{12}x\tan^{17}x\,dx = \int \boxed{u^{11}\left(u^2-1\right)^8}\,du.$

Your answer is a binomial in terms of u raised to a power multiplied by u raised to a power.

Do not multiply it out. Do not find the antiderivative. Just leave it as a polynomial. Hint:

$$\int \sec^{12}x\tan^{17}x\,dx = \int \sec^{11}x\tan^{16}x\sec x\tan x\,dx = \int u^{11}\tan^{16}x\,du = \int u^{11}\left(\tan^2 x\right)^8 du = \int u^{11}\left(\sec^2 x - 1\right)^8 du$$
$$= \int u^{11}\left(u^2-1\right)^8 du$$

3. Consider the integral $\int \frac{\sec^2\theta}{\tan^2\theta}d\theta$.

a. Select which of these is the antiderivative for the integral $\int \frac{\sec^2\theta}{\tan^2\theta}d\theta$.

A. $\sin\theta + C$ B. $\cos\theta + C$ C. $\tan\theta + C$ D. $\csc\theta + C$ E. $\sec\theta + C$ F. $\cot\theta + C$
G. $-\sin\theta + C$ H. $-\cos\theta + C$ I. $-\tan\theta + C$ J. $-\csc\theta + C$ K. $-\sec\theta + C$ (L.) $-\cot\theta + C$
M. All of these. N. None of these.

b. Explain your reasoning for your selection.

$$\int \frac{\sec^2\theta}{\tan^2\theta}d\theta = \int \frac{1}{\tan^2\theta}\sec^2\theta\,d\theta = \int \frac{1}{u^2}du = -\frac{1}{u} + C = -\frac{1}{\tan\theta} + C = -\cot\theta + C$$

4. $\int \cos^3 x\sin^8 x\,dx = \boxed{\tfrac{1}{9}\sin^9 x - \tfrac{1}{11}\sin^{11}x} + C$

Show work. Your final answer should be in terms of a trig function.

Let $u =$ $\sin x$. Then $du =$ $\cos x$ dx

$$\int \cos^3 x\sin^8 x\,dx = \int u^8\cos^2 x\cos x\,dx = \int u^8\cos^2 x\,du = \int u^8(1-\sin^2 x)\,du = \int u^8(1-u^2)\,du = \int (u^8 - u^{10})\,du$$
$$= \tfrac{1}{9}u^9 - \tfrac{1}{11}u^{11} + C = \tfrac{1}{9}\sin^9 x - \tfrac{1}{11}\sin^{11}x + C$$

5. $\displaystyle\int \frac{\tan^2 x}{\sec^2 x}\,dx =$ $\boxed{\frac{1}{2}x \quad - \frac{1}{4}\sin 2x}$ $+ C$

Method 1:

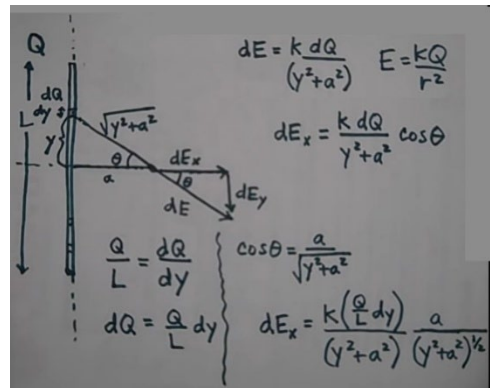
$$\int \frac{\tan^2 x}{\sec^2 x}\,dx = \int \frac{\sin^2 x}{\cos^2 x}\cdot\frac{1}{\sec^2 x}\,dx = \int \frac{\sin^2 x}{\cos^2 x}\cdot\cos^2 x\,dx = \int \sin^2 x\,dx = \frac{1}{2}\int (1-\cos 2x)\,dx$$

$$\frac{1}{2}\int (1-\cos 2x)\,dx \;=\; \frac{1}{2}\int dx \;-\; \frac{1}{2}\cdot\frac{1}{2}\int \cos 2x(2)\,dx = \frac{1}{2}x \;-\frac{1}{4}\left(\sin 2x\right) + C = \frac{1}{2}x \;-\; \frac{1}{4}\sin 2x + C$$

Method 2:

$$\int \frac{\tan^2 x}{\sec^2 x}\,dx = \int \frac{\sec^2 x - 1}{\sec^2 x}\,dx = \int\left(\frac{\sec^2 x}{\sec^2 x} \;-\; \frac{1}{\sec^2 x}\right)dx = \int\left(1 \;-\; \cos^2 x\right)dx = \int \sin^2 x\,dx = \frac{1}{2}\int (1-\cos 2x)\,dx$$

Bonus In PHYS 251 the problem to the right arises. Watch this problem solved [HERE], where he grabs the integral from a table of values and defers the details to your math instructor.
A line of charge Q is spread out in space along the y-axis with length L.
To find the total electric field a distance a from its midpoint, we get an integral that simplifies to the following:



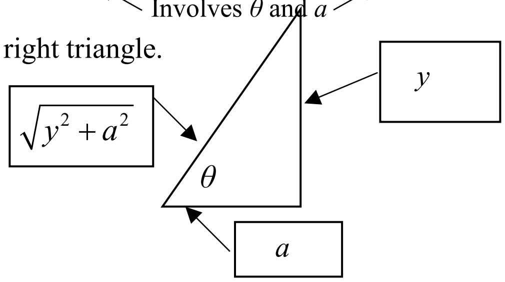
$$E_x = \frac{2akQ}{L}\int_0^{L/2} \frac{1}{\left(y^2+a^2\right)^{3/2}}\,dy$$

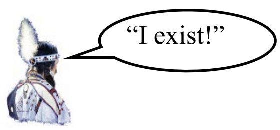
Find $\displaystyle\int \frac{a}{\left(y^2+a^2\right)^{3/2}}\,dy$ using trig substitution. Let $y = a\tan\theta$.

Then dy = $a\sec^2\theta$ $d\theta$ so $\sqrt{y^2+a^2}$ = $a\sec\theta$ $\left(y^2+a^2\right)^{3/2}$ = $(a\sec\theta)^3$

Involves θ and a

Write $\displaystyle\int \frac{1}{\left(y^2+a^2\right)^{3/2}}\,dy$ entirely in terms of θ. Label the right triangle.





$$y = a\tan\theta$$

$$\tan\theta = \frac{OPP}{ADJ} = \frac{y}{a}$$

Involves θ

Simplify your answer in the boxes as much as possible.

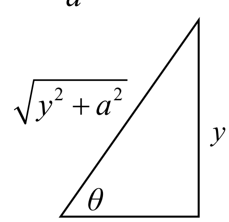
$$\int \frac{1}{\left(y^2+a^2\right)^{3/2}}\,dy = \int \left(\boxed{\frac{1}{a^2}\cos\theta}\right)d\theta = \boxed{\frac{1}{a^2}\sin\theta} + C$$

$$\int \frac{1}{\left(y^2+a^2\right)^{3/2}}\,dy = \int \frac{1}{(a\sec\theta)^3}\,a\sec^2\theta\,d\theta = \int \frac{a\sec^2\theta}{a^3\sec^3\theta}\,d\theta = \frac{1}{a^2}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{a^2}\int \cos\theta\,d\theta = \frac{1}{a^2}\sin\theta + C$$

Write entirely in terms of a and y.

$$\sin\theta = \frac{OPP}{HYP} = \frac{y}{\sqrt{y^2+a^2}}$$

$$\frac{1}{a^2}\sin\theta = \frac{1}{a^2}\frac{y}{\sqrt{y^2+a^2}}$$

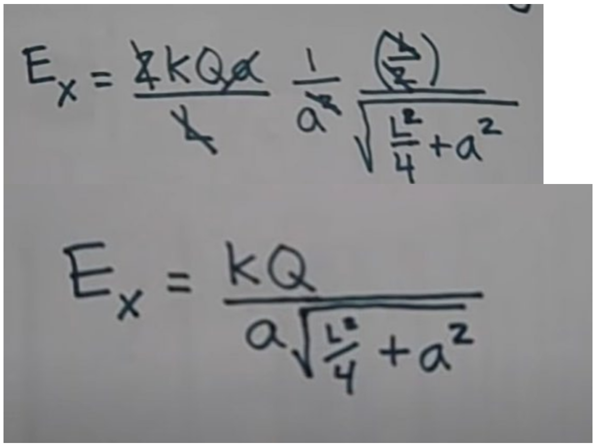


$$\int \frac{a}{\left(y^2+a^2\right)^{3/2}}\,dy = \boxed{\frac{1}{a^2}\frac{y}{\sqrt{y^2+a^2}}} + C$$

To finish the problem, see the next page for the solution to $E_x = \dfrac{2akQ}{L}\displaystyle\int_0^{L/2} \frac{1}{\left(y^2+a^2\right)^{3/2}}\,dy$

To finish the problem, here is the solution:

$$E_x = \frac{2akQ}{L}\int_0^{L/2}\frac{1}{\left(y^2+a^2\right)^{3/2}}\,dy = \frac{2akQ}{L}\,\frac{y}{a^2\sqrt{y^2+a^2}}\Bigg|_{y=0}^{y=L/2} = \frac{2akQ}{La^2}\,\frac{\frac{L}{2}}{\sqrt{(\frac{L}{2})^2+a^2}} = \frac{kQ}{a\sqrt{\frac{L^2}{4}+a^2}}$$



Watch this problem solved HERE.