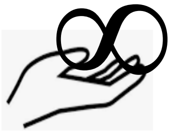
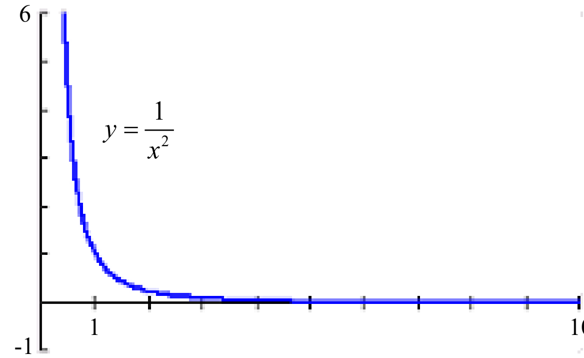


How Improper!



1. Consider the area $A(b)$ under the curve $y = \frac{1}{x^2}$ from $x = 1$ to $x = b$.

a. Without doing any calculations, find the value of $\int_1^1 \frac{1}{x^2}\,dx$.

Enter the value in the table below. Why is this true?

b. Use the FNINT command to complete the other entries in the table.
Split the work among your group.
Report full values with all the digits.

c. Conjecture with your group what to write in the last column.

Endpoint	b	1	100	100,000	50,000,000	∞
Area = $A(b)$	$\int_1^b \frac{1}{x^2}\,dx$					???

d. We can't evaluate expressions at infinity, but we can consider values that approach infinity.

Use correct limit notation to express what you conjectured in the last column. $\lim\limits_{\square} A(b) = \lim\limits_{\square} \int_1^b \frac{1}{x^2}\,dx = \boxed{}$

e. Use the FTC to write an expression that represents the area under $y = \frac{1}{x^2}$ from $x = 1$ to $x = b$.
Your expression involves b. Please simplify.

$A(b) = \int_1^b \frac{1}{x^2}\,dx =$

f. Produce a graph of $A(b)$ with your grapher. Use the window $0 \le x \le 10$ by $-1 \le y \le 6$.
Then sketch $A(b)$ on the above graph.

a. Does this match your conjecture?

b. Complete: The horizontal line $y =$ ___ is a ____________________ for the graph of $y = \frac{1}{x^2}$.

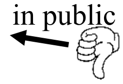
The horizontal line $y =$ ___ is a ____________________ for the graph of $A(b)$.

g. Using limit laws with correct notation show what you claimed in part d will hold up in a court of law and is true.
Use part **e**.

$\lim\limits_{\square} \int_1^b \frac{1}{x^2}\,dx =$

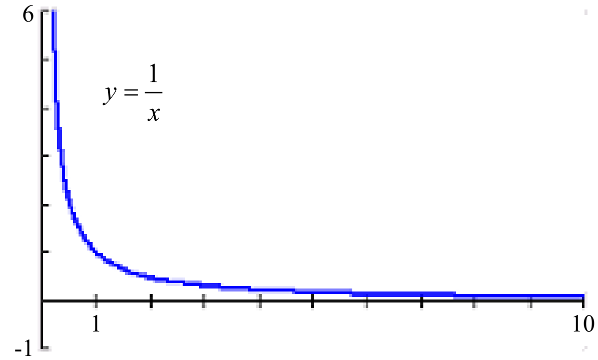
h. Think of a shortcut notation that might make sense to write the expression $\lim\limits_{\square} \int_1^b \frac{1}{x^2}\,dx$

This is formally called an "improper integral". It is perfectly legal notation. However, we never write $\frac{1}{\infty}$ in public since ∞ is not a number. One can not hold ∞ in the palm of one's hand.



i. Use FNINT with caution. Explore what happens if you try using FNINT with $b = 10^8$ for $A(b) = \int_1^b \frac{1}{x^2}\,dx$.
$A(b) \approx$ ____
This can occur with all computational software and is called a "stack overflow" error.

2. Consider the area $A(b)$ under the curve $y = \frac{1}{x}$ from $x = 1$ to $x = b$.



a. Without doing any calculations, find the value of $\int_1^1 \frac{1}{x}\,dx$.

Enter the value in the table below.

b. Use the FNINT command to complete the other entries in the table. Split the work among your group. Report to two decimal places.

c. Conjecture with your group what to write in the last column.

Endpoint	b	1	50,000,000	$1{,}000{,}000{,}000 = 10^9$	$100{,}000{,}000{,}000 = 10^{11}$	∞
Area = $A(b)$	$\int_1^b \frac{1}{x}\,dx$					???

d. Complete the box with what you conjectured in the last column. $\int_1^\infty \frac{1}{x}\,dx = \lim_{b\to\infty} A(b) = \lim_{b\to\infty} \int_1^b \frac{1}{x}\,dx = \boxed{}$

e. Use the FTC to write an expression in the box that represents the area under $y = \frac{1}{x}$ from $x = 1$ to $x = b$.

Your expression involves b. Please simplify as much as possible.

$A(b) = \int_1^b \frac{1}{x}\,dx =$ $\qquad$ $A(b) = \boxed{}$

f. Produce a graph of $A(b)$ with your grapher. Use the window $0 \le x \le 10$ by $-1 \le y \le 6$.
Then sketch $A(b)$ on the above graph.
a. Does this match your conjecture?

b. Complete: The horizontal line $y =$ ___ is a ____________________________ for the graph of $y = \frac{1}{x}$.

The graph of $A(b) = \boxed{}$ ________________ a horizontal asymptote.
{has, does not have }

g. Using limit laws with correct notation show what you claimed in part **2d** will hold up in a court of law and is true. Use part **e**.

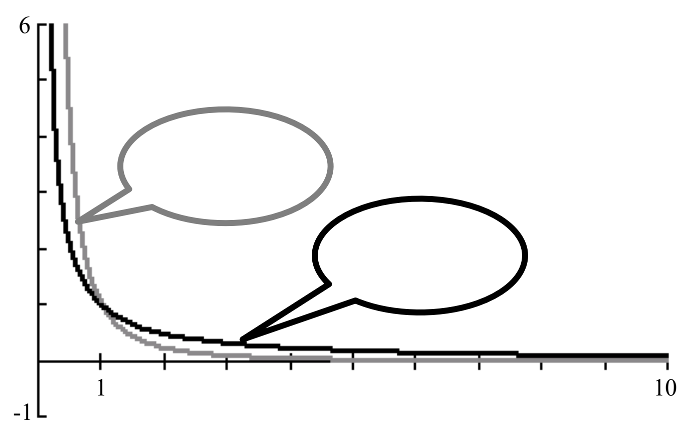
$\lim_{b\to\infty} \int_1^b \frac{1}{x}\,dx =$

h. Optional: Suppose the upper limit $b = e^{21}$. Based on part **e**, what do you think $\int_1^{e^{21}} \frac{1}{x}\,dx$ would be? Explain. Check with FNINT.

3. Consider the functions $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$ in Quadrant I.

a. Label each graph with its formula.

b. How can you tell which is which by inspection?



c. How might your answer in **3b** give you intuition about your conclusions in **1d** and **2d**?