Fruit from a Fertile Father

1. Suppose $f(w) = \sum\limits_{n=0}^{\infty} w^n = 1 + w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7 + w^8 + w^9 + w^{10} + w^{11} + w^{12} + w^{13} + w^{14} + w^{15} + \ldots$

   a. If $w = -1$, then $f(-1) = \sum\limits_{n=0}^{\infty}(-1)^k = 1 + \boxed{} + \boxed{} + \boxed{} + \boxed{} + \ldots = \boxed{}$ Write an exact answer, $\infty, -\infty$, or DNE

   If $w = 1$, then $f(-1) = \sum\limits_{n=0}^{\infty}(1)^k = 1 + \boxed{} + \boxed{} + \boxed{} + \boxed{} + \ldots = \boxed{}$ Write an exact answer, $\infty, -\infty$, or DNE

   b. We have $f(w) = \sum\limits_{n=0}^{\infty} w^n = 1 + w + w^2 + w^3 + w^4 + \ldots$ converges to the function $g(w) = \dfrac{1}{\boxed{}}$ for $\boxed{} < w < \boxed{}$

   because _____

   Hint: Refer to the title of this activity. What other convergence tests can also help?

   **c.** Pick the best answer: The interval reported in part **b** is called the ***interval*** of _____
   { convergence, conviction, confluence, concupiscence, concubinage, conception, conveyorization, confederation, constitution }

   d. Sketch the interval reported in part **b** on a number line: ⟵————————|————————⟶
   0

   e. Which of the following is the graph of $f(w) = \sum\limits_{n=0}^{\infty} w^n = 1 + w + w^2 + w^3 + w^4 + \ldots$ over the interval in part **b**?

   Use part **a** (and not a grapher). Circle your selection and enter numbers in the boxes for the choice you circled.

   A.   B.   C.   D.

   E.   F.   G.   H.
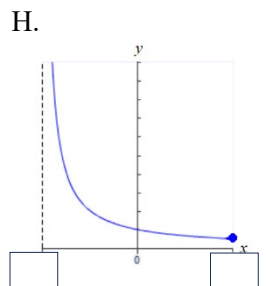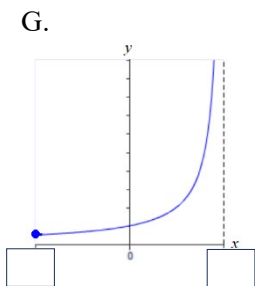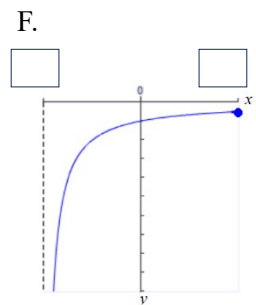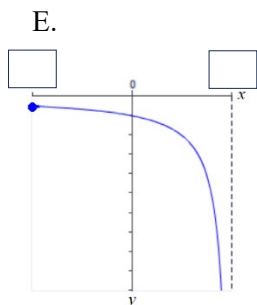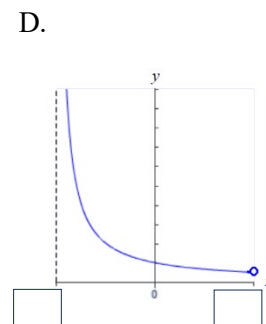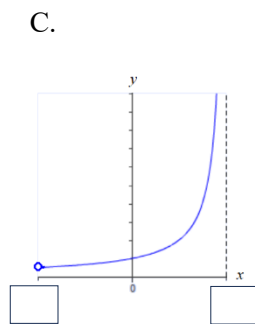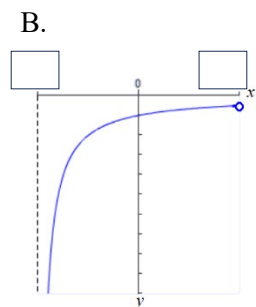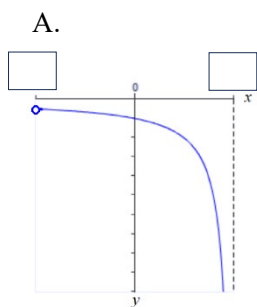
   f. The ***center*** of the interval in part **b** is $w =$ _____ . The ***radius*** of _____ is $R = \boxed{}$.
   {same answer in part c}

2. Discuss the relationship between the graphs of $f(w)$ in part **e** and your answers to part **1a**.
   Discuss the relationship between the series $f(w)$ and the function $g(w) = 1/(1-w)$. What is the same? What is different?

3. Suppose $a(x) = \sum_{n=0}^{\infty} -100\left(\frac{x-20}{5}\right)^n = -100 - 100\left(\frac{x-20}{5}\right) - 100\left(\frac{x-20}{5}\right)^2 - 100\left(\frac{x-20}{5}\right)^3 - 100\left(\frac{x-20}{5}\right)^4 - \dots$

a. Report the interval of convergence. Show work.

$\boxed{} < x < \boxed{}$

b. The *center* of the interval of convergence is $x = $ _____ . The *radius* of convergence is $R = \boxed{}$.

c. If $x = \boxed{}$, the **left** endpoint of the interval of convergence, then

$f(\boxed{}) = \sum_{n=0}^{\infty} -100(\boxed{})^k = -100 + \boxed{} - \boxed{} + \boxed{} - \boxed{} + \dots = \boxed{}$ Write an exact answer, $\infty, -\infty$, or DNE

If $x = \boxed{}$, the **right** endpoint of the interval of convergence, then

$f(\boxed{}) = \sum_{n=0}^{\infty} -100(\boxed{})^k = -100 - \boxed{} - \boxed{} - \boxed{} - \boxed{} - \dots = \boxed{}$ Write an exact answer, $\infty, -\infty$, or DNE

d. Which of the following is equivalent to $a(x) = \sum_{n=0}^{\infty} -100(\frac{x-20}{5})^k = -100 - 100(\frac{x-20}{5}) - 100(\frac{x-20}{5})^2 - 100(\frac{x-20}{5})^3 - \dots$ on its interval of convergence? Select one.

A. $\dfrac{500}{25-x}$ B. $\dfrac{500}{25+x}$ C. $\dfrac{500}{x-25}$ D. $\dfrac{100}{25-x}$ E. $\dfrac{100}{25+x}$ F. $\dfrac{100}{x-25}$ G. $\dfrac{500}{15-x}$ H. $\dfrac{500}{15+x}$ I. $\dfrac{500}{x-15}$

e. Which of the following is the graph of $a(x) = \sum_{n=0}^{\infty} -100(\frac{x-20}{5})^k = -100 - 100(\frac{x-20}{5}) - 100(\frac{x-20}{5})^2 - 100(\frac{x-20}{5})^3 - \dots$ on its interval of convergence?

Circle your selection and enter numbers in the boxes for the choice you circled.

A.

B.

C.

D.

E.

F.

G.

H.
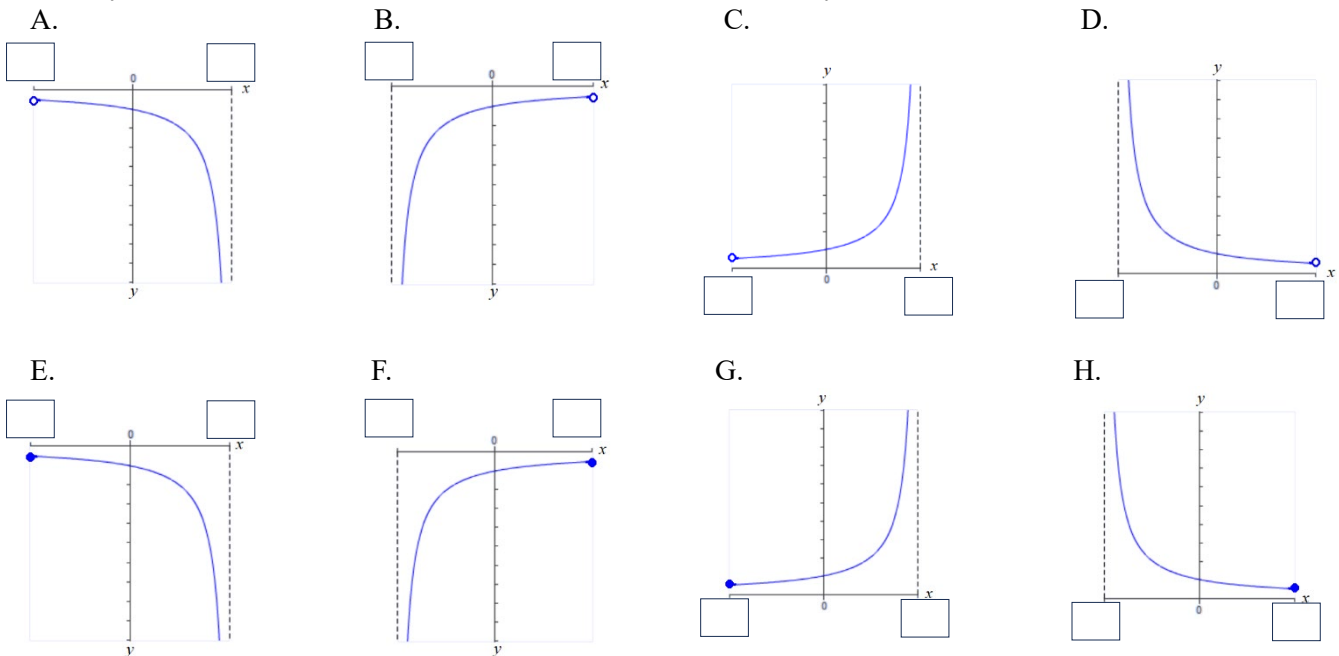
f. Report the equation of the vertical asymptote. _____   Report the coordinates of the hole: ($\boxed{}$, $\boxed{}$)

5. Discuss the relationship between the graph of $a(x) = \sum_{n=0}^{\infty} -100(\frac{x-20}{5})^k = -100 - 100(\frac{x-20}{5}) - 100(\frac{x-20}{5})^2 - 100(\frac{x-20}{5})^3 - \dots$

in part **3e** and its father function, $f(w) = \sum_{n=0}^{\infty} w^n = 1 + w + w^2 + w^3 + w^4 + \dots$ shown in part **1e**.