

Rhino Bonus: Euler's Famous Discovery at Age 28

The infinite series of the reciprocals of the squares of positive integers was known to converge, but no one could find the exact value until 1735, when Euler, 28 years old, showed: $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{6}$.

The mathematical world rejoiced. See below why this discovery was a gift that kept on giving.

(+0.5) **a.** Use the fact that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{6}$ to find the exact value of $\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \frac{1}{8^2} + \dots$

Show work below.

(+0.5) **b.** Use the fact that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{6}$ and part **a** to find the exact value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$

Show work below.