9.1 and 9.2 Modeling Differential Equations and Verifying Solutions

Important Ideas:

1. Which of the following functions are solutions to the differential equation $y'' + y = 0$? (Choose all that apply.)

   A) $y = e^x$

   B) $y = e^{-x}$

   C) $y = \sin x$

   D) $y = -\cos x$

   E) $y = 3\cos(x)$

2. The number of fruit flies increases at a rate proportional to its current population, *F*. Write a differential equation to represent this situation.

3. Find the general solution to each differential equations

   a. $x\dfrac{dy}{dx} = 5$

   b. $x\dfrac{dy}{dx} = 5\ln x$

To find the general solution to $\dfrac{dy}{dt} = k(y - A)$, assuming $y - A \neq 0$, we perform three steps. Fill in the blanks.

1. Separate: $\dfrac{dy}{dt} = k(y-A)$ — Separate the variables so y is on one side, t is on the other by multiplying and dividing.

$$\frac{dy}{(y-A)} = kdt$$

2. Integrate: $\displaystyle\int \frac{dy}{(y-A)} = \int kdt$ — Integrate both sides. Since $y - A \neq 0$, $\displaystyle\int \frac{dy}{(y-A)} = \ln|y-A| + C_1$

$$\ln|y-A| + C_1 = kt + C_2$$

We *could* include the constant of integration on both sides or not,
and just write the following, assuming C_3 = ________

$$\ln|y-A| = kt + C_3$$

3. Isolate: $\ln|y-A| = kt + C_3$ — If possible, get y all by itself.

$e^{\ln|y-A|} = e^{kt+C_3}$ — Make both sides a power of e.

$|y-A| = e^{C_3}e^{kt}$ — Use an inverse property and law of exponents

$|y-A| = C_4e^{kt}$ — What are we assuming here? C_4 = ________

$y - A = \pm C_4e^{kt}$ — Remove absolute value signs.

$y - A = Ce^{kt}$ — What are we assuming here? C = ________

$y = Ce^{kt} + A$ — Subtract A from both sides to solve for y.

1. Solve the differential equation $\dfrac{dy}{dt} = k(y - A)$ for the special case when $y - A = 0$.

2. Suppose Renfield brings with him a glass of ice water to the wine cellar, sets it down to measure Sherry's body temperature, and then leaves the non-empty glass in the cellar.

a. Use the above template to report the **general solution** for $\dfrac{dy}{dt} = k(y-60)$, where y' is the rate the ice water warms.

y = ______________________________ + 60

b. Assume the initial temperature of the ice water is 32˚F. If two hours later it warms to 42˚F, find the **particular solution** to **2a**. Report k to 2 decimal places. Use technology or algebra.

3. Fill in the boxes.
Recall what C represents graphically for the family $y = Ce^{kt} + 60$.