Commonly Used Taylor Series

SERIES			RADIUS OF CONVERGENCE	WHEN IS VALID/TRUE
$\frac{1}{1-x}$	$=$	$1 + x + x^2 + x^3 + x^4 + \dots$		NOTE THIS IS THE GEOMETRIC SERIES. JUST THINK OF x AS r
	$=$	$\sum_{n=0}^{\infty} x^n$		$x \in (-1, 1)$ $\quad -1 < x < 1$
e^x	$=$	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$		$e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$ $e^{(17x)} = \sum_{n=0}^{\infty} \frac{(17x)^n}{n!} = \sum_{n=0}^{\infty} \frac{17^n x^n}{n!}$
	$=$	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$		$x \in \mathbb{R}$ $\quad -\infty < x < \infty$
$\cos x$	$=$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$		NOTE $y = \cos x$ IS AN <u>EVEN</u> FUNCTION (I.E., $\cos(-x) = +\cos(x)$) AND THE TAYLOR SERIS OF $y = \cos x$ HAS ONLY <u>EVEN</u> POWERS.
	$=$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$		$x \in \mathbb{R}$ $\quad -\infty < x < \infty$
$\sin x$	$=$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$		NOTE $y = \sin x$ IS AN <u>ODD</u> FUNCTION (I.E., $\sin(-x) = -\sin(x)$) AND THE TAYLOR SERIS OF $y = \sin x$ HAS ONLY <u>ODD</u> POWERS.
	$=$	$\sum_{n=1}^{\infty} (-1)^{(n-1)} \frac{x^{2n-1}}{(2n-1)!} \overset{\text{or}}{=} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$		$x \in \mathbb{R}$ $\quad -\infty < x < \infty$
$\ln(1+x)$	$=$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$		QUESTION: IS $y = \ln(1+x)$ EVEN, ODD, OR NEITHER?
	$=$	$\sum_{n=1}^{\infty} (-1)^{(n-1)} \frac{x^n}{n} \overset{\text{or}}{=} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$		$x \in (-1, 1]$ $\quad -1 < x \le 1$ Integrate terms of geometric series $\frac{1}{1-x}$ and perform a substitution.
$\tan^{-1} x$	$=$	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots$		QUESTION: IS $y = \arctan(x)$ EVEN, ODD, OR NEITHER?
	$=$	$\sum_{n=1}^{\infty} (-1)^{(n-1)} \frac{x^{2n-1}}{2n-1} \overset{\text{or}}{=} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$		$x \in [-1, 1]$ $\quad -1 \le x \le 1$

Perform a substitution of $-x^2$ in the geometric series $\frac{1}{1-u}$ and integrate.
Check this by differentiation of the series for arctan x and decomposing.