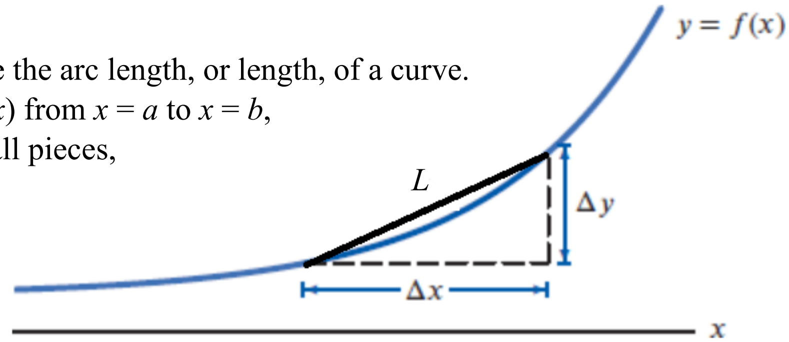
Arc Length

A definite integral can be used to compute the arc length, or length, of a curve. To compute the length of the curve $y = f(x)$ from $x = a$ to $x = b$, where $a < b$, we divide the curve into small pieces, each one approximately straight.

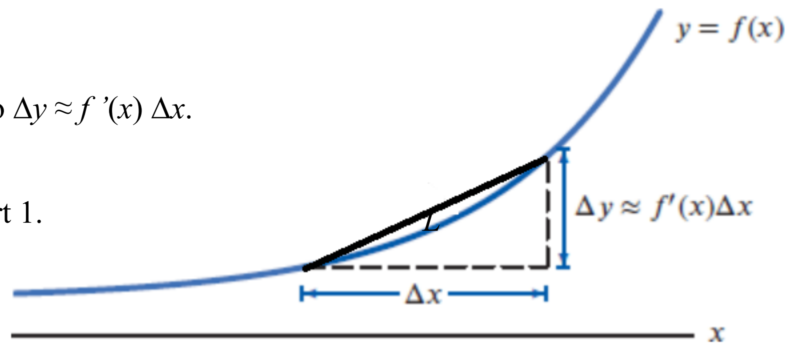
Assume f and f' are continuous on $[a, b]$.



1. We enlist the help of a very old dead guy who smells like Greek Feta cheese: 
   What does he say is true about length of the hypotenuse L?

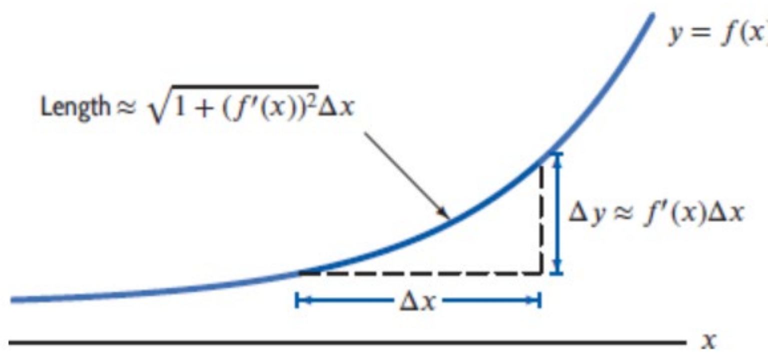
2. We approximate $f'(x) = \dfrac{dy}{dx} \approx \dfrac{\Delta y}{\Delta x}$ so $\Delta y \approx f'(x)\,\Delta x$.

   Substitute this in your formula in part 1.



3. Explain why, as the number of subintervals increases without bound, we have

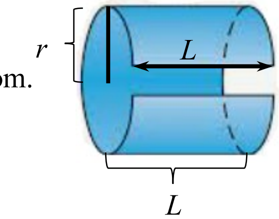
   $$\text{Length} = \int_a^b \sqrt{1+(f'(x))^2}\,dx = \int_a^b \sqrt{1+(y')^2}\,dx$$



Similarly, if $x = g(y)$ and g' are continuous on $c \le y \le d$, we interchange the roles of x and y:

$$\text{Length} = \int_c^d \sqrt{1+(g'(y))^2}\,dy = \int_c^d \sqrt{1+\left(\tfrac{dx}{dy}\right)^2}\,dy$$

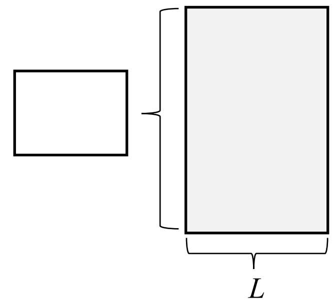
Lateral Surface Area



The lateral (side) surface area of a solid of revolution includes only its side, not its top or bottom. For example, the lateral surface area of a soup can is the area of the label.

1. Complete the boxes to report the lateral surface area of a soup can with radius r and length L from top to bottom.

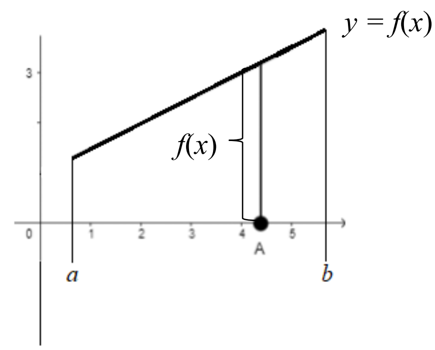
   Lateral Surface Area = $\square$



2. Report the **arc length *L*** of any function $y = f(x)$ from $x = a$ to $x = b$.

$$L = \int_{\square}^{\square} \left(\qquad \right) d\square$$

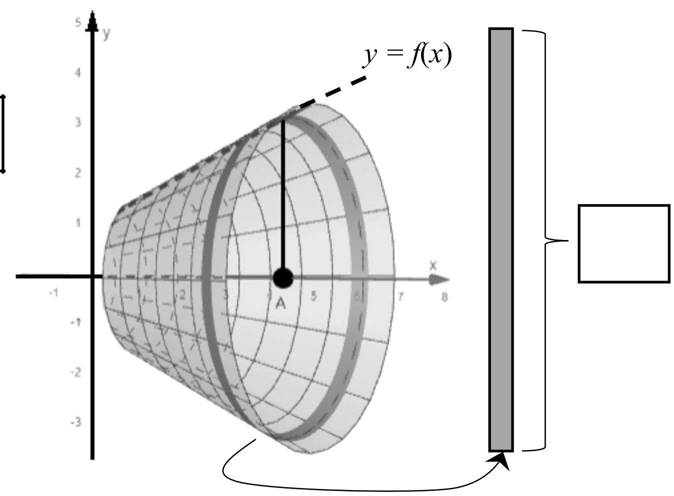
Decide x or y



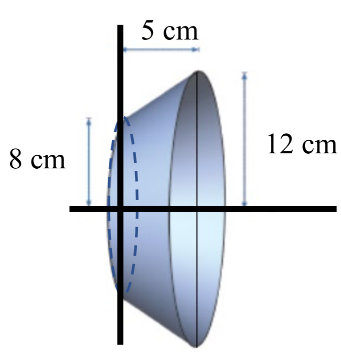
3. Report the **lateral surface area LSA** created by revolving any continuous, differentiable function $y = f(x)$ about the x-axis from $x = a$ to $x = b$, assuming $f'(x)$ is continuous.

$$LSA = \int_{\square}^{\square} \left(\qquad \right) d\square$$

Decide x or y



4. Heinz is shown below with his water dish.
   Use the FTC to set up and find its *exact* **lateral surface area**.
   Use the FTC to find the *exact* **volume**.



Cool fact: The **lateral surface area LSA** created by revolving any continuous, differentiable function $x = g(y)$ about the y-axis from $x = c$ to $x = d$, assuming $g'(y)$ is continuous would be

$$\int_c^d 2\pi g(y)\sqrt{1+\left(g'(y)\right)^2}\,dy = \int_c^d 2\pi g(y)\sqrt{1+\left(\tfrac{dx}{dy}\right)^2}\,dy$$

Rhino Participation Bonus (+2): Set up a definite integral and use the FTC to show the LSA of the cone is LSA = πrL and the volume is $V = \frac{1}{3}\pi r^2 h$.

You can integrate with respect to x or y.