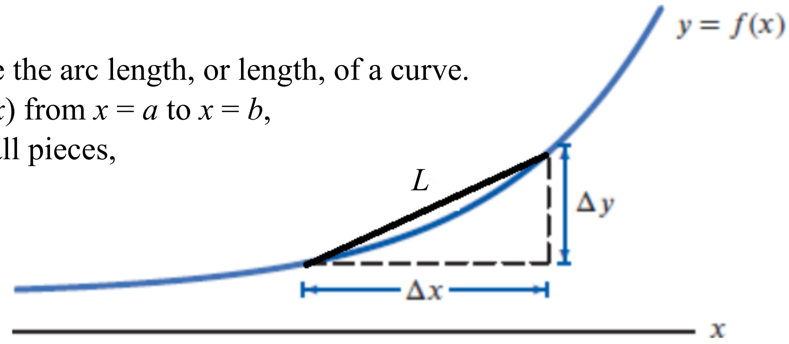
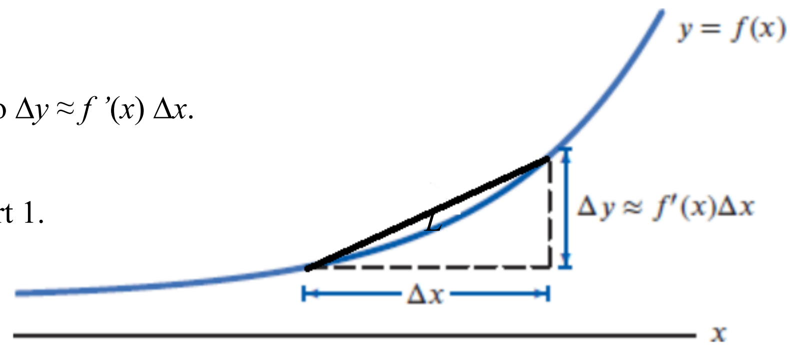
Arc Length



A definite integral can be used to compute the arc length, or length, of a curve. To compute the length of the curve $y = f(x)$ from $x = a$ to $x = b$, where $a < b$, we divide the curve into small pieces, each one approximately straight.

Assume f and f' are continuous on $[a, b]$.

1. We enlist the help of a very old dead guy who smells like Greek Feta cheese: 
   What does he say is true about length of the hypotenuse L?

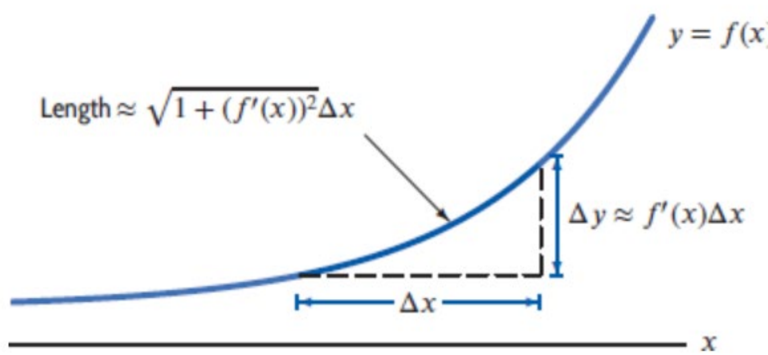


2. We approximate $f'(x) = \dfrac{dy}{dx} \approx \dfrac{\Delta y}{\Delta x}$ so $\Delta y \approx f'(x)\,\Delta x$.

   Substitute this in your formula in part 1.

3. Explain why, as the number of subintervals increases without bound, we have

   $$\text{Length} = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$
   $$= \int_a^b \sqrt{1+(y')^2}\,dx$$



Similarly, if $x = g(y)$ and g' are continuous on $c \le y \le d$, we interchange the roles of x and y:

$$\text{Length} = \int_c^d \sqrt{1+(g'(y))^2}\,dy = \int_c^d \sqrt{1+\left(\tfrac{dx}{dy}\right)^2}\,dy$$