A Very Famous Limit

(+.25) 1. Complete the boxes and blanks to show the amazing fact that $\lim\limits_{n\to\infty}\left(1+\frac{r}{n}\right)^n = e^r$.

(+.25) 2. Show that $\lim\limits_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt} = Pe^{rt}$.

1. Let $L = \lim\limits_{n\to\infty} \ln\left(1+\frac{r}{n}\right)^n$ and find L. TIP: The contents of each of the dashed gray boxes is the same.

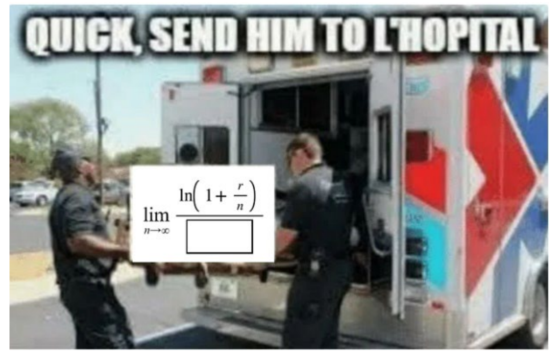
If $L = \lim\limits_{n\to\infty} \ln\left(1+\frac{r}{n}\right)^n$ use a property of logarithms to write

$$L = \lim_{n\to\infty}\left(\square \ln\left(1+\frac{r}{n}\right)\right) = \lim_{n\to\infty}\frac{\ln\left(1+\frac{r}{n}\right)}{\square}$$

which will be in

the *Indeterminate Form* of ____________________.

$\left\{\frac{\infty}{\infty}, \frac{0}{0}, 0^0, 0\cdot\infty, \infty-\infty, 1^\infty, \infty^0\right\}$



Using L'Hôpital's rule we write $L = \lim\limits_{n\to\infty}\dfrac{\frac{d}{dn}\ln\left(1+\frac{r}{n}\right)}{\frac{d}{dn}\square} = \lim\limits_{n\to\infty} \square \cdot \dfrac{r\cdot\frac{d}{dn}\square}{\frac{d}{dn}\square}$ (1)

$$= \square \cdot r = \square$$

Since $\lim\limits_{n\to\infty}\ln\left(1+\frac{r}{n}\right)^n = \ln\lim\limits_{n\to\infty}\left(1+\frac{r}{n}\right)^n$, we have the cool fact that

$$\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n = e^{\ln\lim\limits_{n\to\infty}\left(1+\frac{r}{n}\right)^n} = e^{\lim\limits_{n\to\infty}\ln\left(1+\frac{r}{n}\right)^n} = e^L = \square$$

2. Using correct limit notation, show $\lim\limits_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt} = Pe^{rt}$.