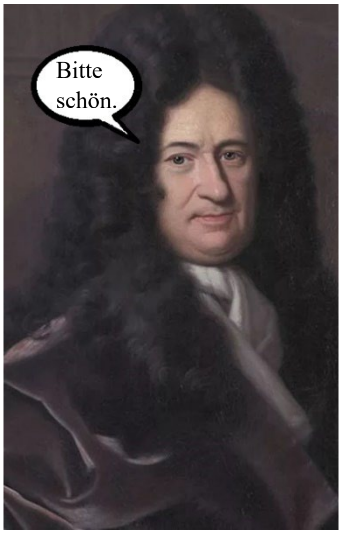


A Gift from Gottfried (and Big Daddy)

+1 Rhino Bonus Participation Point

BIG DADDY

1. Suppose $f(w) = \sum_{n=0}^{\infty} w^n = 1 + w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7 + w^8 + w^9 + w^{10} + w^{11} + \ldots$

a. Replace w with $-x^2$ in the above expression:

$$f(-x^2) = \sum_{n=0}^{\infty} \left(\square\right)^n = 1 + \square + \left(\square\right)^2 + \left(\square\right)^3 + \left(\square\right)^4 + \ldots$$

b. Let's call the above child series $d(x)$. Simplify.

$$d(x) = 1 + \square + \square + \square + \square + \ldots$$

Write the expression in sigma notation so it is simplified. $d(x) = \sum_{n=0}^{\infty} \square$

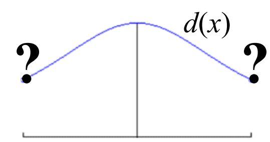
c. Report the radius of convergence of $d(x)$. $R = \square$

d. Complete: the **left** endpoint of the interval of convergence is $x =$ ______ and is ______________ in the interval.
Justify your claim in the space below. { included, excluded}

e. Complete: the **right** endpoint of the interval of convergence is $x =$ ______ and is ______________ in the interval.
Justify your claim in the space below. { included, excluded}

f. Write the interval of convergence in inequality notation. ___________________________________

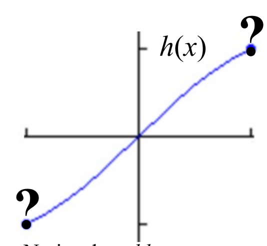
g. Within its interval of convergence, the series $d(x)$ is equivalent to the function $d(x) = \dfrac{1}{\square}$.



Notice the *even* symmetry (about the *y-axis*)

2. Integrate each term of $d(x)$ to create a new series $h(x) = \int d(x)\,dx$. No need for a "+ C".

a. $h(x) = \int d(x)\,dx = \square + \square + \square + \square + \square + \ldots$



Notice the *odd* symmetry (about the *origin*)

Write the expression for the series $h(x)$ in sigma notation so it is simplified. $h(x) = \sum_{n=0}^{\infty} \square$

Hint: Integrate your expressions in **1b**.

b. Report the radius of convergence of $h(x)$. $R =$ _____

c. The **left** endpoint of the interval of convergence is $x =$ ______ and is ______________ in the interval.
Justify your claim in the space below. { included, excluded}

$$h(\square) = \square + \square - \square + \square - \square + \ldots = \sum_{n=0}^{\infty} \square$$

d. The **right** endpoint of the interval of convergence is $x =$ ______ and is ______________ in the interval.
Justify your claim in the space below. { included, excluded}

$$h(\square) = \square + \square - \square + \square - \square + \ldots = \sum_{n=0}^{\infty} \square$$

e. Write the interval of convergence in inequality notation. ___________________________________

f. Within its interval of convergence, the series $h(x)$ is equivalent to the function $h(x) = \int d(x)\,dx =$ __________

g. If we make either substitution of $x = 1$ in the series $h(x)$, we have the remarkable result that was published in the 17th century by Gottfried Leibniz. Enter exact numerical values in each box.

$$\square + \square + \square + \square + \square + \square + \square + \square + \ldots = \square$$