A Gift from Big Daddy

BIG DADDY

1. Suppose $f(w)=\sum_{n=0}^{\infty} w^n = 1+w+w^2+w^3+w^4+w^5+w^6+w^7+w^8+w^9+w^{10}+w^{11}+w^{12}+w^{13}+w^{14}+w^{15}+\ldots=\dfrac{1}{1-w}$

a. Replace w with $-x$ in the above expression:

$$f(-x)=\sum_{n=0}^{\infty}(\square)^n = 1+\square+(\square)^2+(\square)^3+(\square)^4+\ldots$$

b. Let's call the above child series $b(x)$. Simplify.

$$b(x)=1+\square+\square+\square+\square+\ldots$$

Write the expression in sigma notation so it is simplified. $b(x)=\sum_{n=0}^{\infty}$ $\square$

c. Report the radius of convergence of $b(x)$. $R=\square$

d. Complete: the **left** endpoint of the interval of convergence is $x=$ _____ and is _____ in the interval.
{ included, excluded}
Justify your claim in the space below.

$b(\square)=\sum_{n=0}^{\infty}(\square)^n = 1+\square+\square+\square+\square+\square+\ldots=\square$ Write an exact answer, ∞,$-\infty$, or DNE $b(x)$

Complete: the **right** endpoint of the interval of convergence is $x=$ _____ and is _____ in the interval.
{ included, excluded}
Justify your claim in the space below.

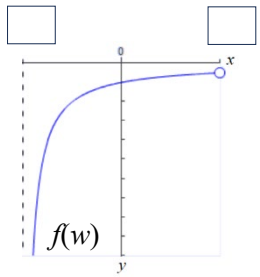
$b(\square)=\sum_{n=0}^{\infty}(\square)^n = 1+\square+\square+\square+\square+\square+\ldots=\square$ Write an exact answer, ∞,$-\infty$, or DNE

e. Write the interval of convergence in inequality notation. _____

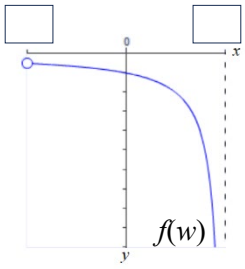
f. Within its interval of convergence, the series $b(x)$ is equivalent to the function $b(x)=\dfrac{1}{\square}$

g. Which of the following is the graph of $b(x)$ over the interval in part **e**?
Use part **d** (and not a grapher). Circle your selection and enter numbers in the boxes for the choice you circled.
The dashed line is a vertical asymptote.

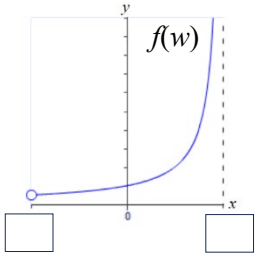
A.



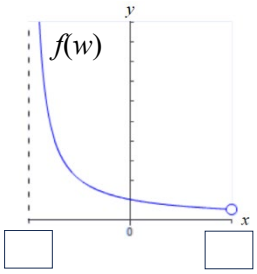
B.



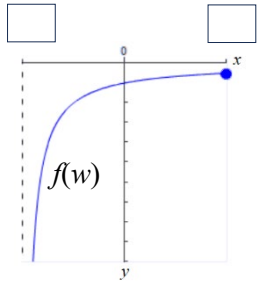
C.



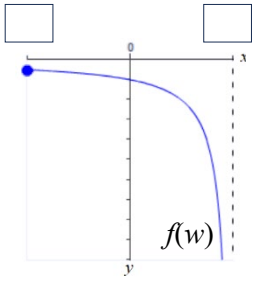
D.



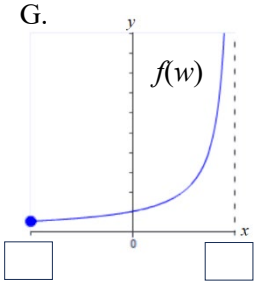
E.



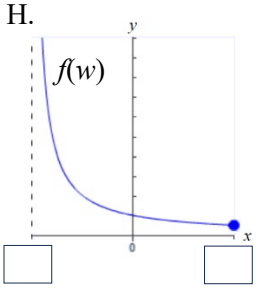
F.



G.



H.



2. On the interval of convergence, the graph of $b(x)$ is a _____ reflection of the graph of $f(w)=\dfrac{1}{1-w}$.
{ horizontal, vertical }

3. Integrate each term of $b(x)$ to create a new series $c(x)=\int b(x)dx$. No need for a "+ C".

a. $c(x)=\int b(x)dx = \square + \square + \square + \square + \square + \ldots$

b. Write the expression for the series $c(x)$ in sigma notation so it is simplified.

c. Report the radius of convergence of $c(x)$. $R =$ _____

d. The **left** endpoint of the interval of convergence is $x =$ _____ and is _____ in the interval.
{ included, excluded}
Justify your claim in the space below.

$c(\square) = \square + \square + \square + \square + \square + \ldots = \sum^{\infty}$ ▭

e. The **right** endpoint of the interval of convergence is $x =$ _____ and is _____ in the interval.
{ included, excluded}
Justify your claim in the space below.

$c(\square) = \square + \square + \square + \square + \square + \ldots = \sum^{\infty}$ ▭

f. Write the interval of convergence in inequality notation. _____

g. Within its interval of convergence, the series $c(x)$ is equivalent to the function $c(x)=\int b(x)dx =$_____

Hint: Integrate your expression in **1f**.

h. If we make either substitution of $x = 1$ in the series $c(x)$, we have the remarkable result below:

$\square + \square + \square + \square + \square + \ldots = \square$

4. Suppose a new (child) power series is made by *differentiating* or *integrating* the original parent.
Discuss if the **center** of the interval of convergence of the child power series might change. (YES / NO)

Discuss if the **radius** of convergence of the child power series might change. (YES / NO)

Discuss if the **interval** of convergence of the child power series might change. (YES / NO)

5. Suppose a new (child) power series is made by *a substitution involving a shift, stretch, shrink or reflection*.
Discuss if the **center** of the interval of convergence of the child power series might change. (YES / NO)

Discuss if the **radius** of convergence of the child power series might change. (YES / NO)

Discuss if the **interval** of convergence of the child power series might change. (YES / NO)