Ruh- Rho! Density as a Function

Examples of Density – Different measurements are possible.

1. The **density of a substance** (e.g. stone, air, wood, or metal) is the mass of a unit volume of the substance and is measured in, say, *grams per cubic centimeter*.
2. **Population density** is measured in, say, *people per mile* (along the edge of a road), or *people per unit area* (in a city), or *bacteria per cubic centimeter* (in a test tube).

Densities of substances with (approximately) the same mass does not need calculus.

1. Complete the table.

Dimensions in cm	Volume, cu. cm	Mass, g	Density ρ
		126	
		3.78	

When the substance or population varies in quantity, it is time to accumulate and call upon our hero!

How do we calculate the total population or total mass if the density is not constant over a region?

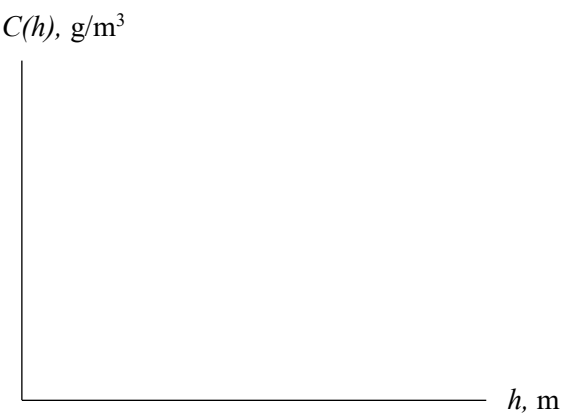
Strategy: Divide the region into small pieces in such a way that the density is approximately constant on each piece, and add the contributions of the pieces.

2. The number of people living next to a certain road varies as it gets farther from the city. Suppose that, x miles out of town, the population density adjacent to the road is given by the population density function $f(x) = 1400e^{-0.7x}$ people/mile. Express the total population living next to the road within tis 6 mile stretch of road.

3. A cylindrical well has a radius of 2.5 meters. It is 9 meters deep.

The concentration of silt in a well is 5 $\frac{\text{g}}{\text{m}^3}$ at the surface ($h = 0$)

and **increases linearly** to 22.1 $\frac{\text{g}}{\text{m}^3}$ at the bottom ($h = 9$).



a. Let $C(h)$ be the silt concentration, in g/m^3, at depth h meters.
The graph of $C(h)$ is an **increasing linear function**.
Report the formula for $C(h)$.

$C(h) =$ _____

b. Set up an integral to calculate the total amount of silt in the well.

$$\int_{\square}^{\square} \left(\qquad \right) dh$$

c. Evaluate your integral and report the total amount of silt to the nearest gram.

The total amount of silt in the 9 meter deep well is _____ grams.

4. Find the mass of the thin 6 m long bar with density (in kg per m) given by $\rho(x) = \begin{cases} 5 & , \ 0 \le x \le 2 \\ x + 5 & , \ 2 < x \le 6 \end{cases}$