We are asked to find $\lim_{x\to 0} (344x + \cos x)^{\frac{7}{8x}}$

1. First find $\lim_{x\to 0} \ln (344x + \cos x)^{\frac{7}{8x}}$

   Use the Bob Barker Property:

   $$\ln (344x + \cos x)^{\frac{7}{8x}} = \frac{7}{8x} \ln (344x + \cos x) = \frac{7\ln (344x + \cos x)}{8x}.$$

   $\lim_{x\to 0} \frac{7\ln (344x + \cos x)}{8x}$ is of the form $\frac{0}{0}$ since $\ln (0 + 1) = 0$. Apply L'Hopital.

   $$\lim_{x\to 0} \frac{7\ln (344x + \cos x)}{8x} \underset{\text{LH}}{=} \lim_{x\to 0} \frac{7 \cdot \frac{d}{dx}(\ln(344x + \cos x))}{\frac{dx}{dx} 8x} = \lim_{x\to 0} \frac{7 \cdot \frac{1}{(344x + \cos x)} \cdot (344 + \sin x)}{8}$$

   $$= \lim_{x\to 0} \frac{7(344 + \sin x)}{8(344x + \cos x)} = \frac{7(344 + 0)}{8(0 + 1)}$$

   $$= \frac{7(344 + 0)}{8(0 + 1)} = 301.$$

2. Use the Cool Facts that $A = e^{\ln A}$, that $\lim_{x\to 0} \ln (344x + \cos x)^{\frac{7}{8x}} = 301$, and a property of limits:

   $$\lim_{x\to \infty} (344x + \cos x)^{\frac{7}{8x}} = \lim_{x\to \infty} e^{\ln (344x + \cos x)^{\frac{7}{8x}}} = e^{\lim_{x\to 0} \ln (344x + \cos x)^{\frac{7}{8x}}} = \boxed{e^{301}}.$$