Find $\lim_{x \to 0} (1 - 1234x)^{\frac{10}{x}}$

1. Let $w = \frac{10}{x}$. Use substitution to rewrite everything in terms of w.
   a. We have $x = \frac{10}{w}$. The expression $1234x$ is $1234 \cdot \frac{10}{w} = \frac{12340}{w}$ so we have

   $$(1 - 1234x)^{\frac{10}{x}} = \left(1 - \frac{12340}{w}\right)^w$$

   b. As $x \to 0$, we have $w = \frac{10}{x} \to \infty$. Observe a graph here to see.
   (Large numbers in the denominator mean very small outputs.)

   c. Use parts a and b to write $\lim_{x \to 0} (1 - 1234x)^{\frac{10}{x}} = \lim_{w \to \infty} \left(1 - \frac{12340}{w}\right)^w$

2. Cool Fact! $\lim_{n \to \infty} \left(1 + \frac{r}{n}\right)^n = e^r$, so $\lim_{w \to \infty} \left(1 - \frac{12340}{w}\right)^w = \lim_{w \to \infty} \left(1 + \frac{-12340}{w}\right)^w = \boxed{e^{-12340}}$.

Another Approach: Use L'Hopital's Rule by setting $y = (1 - 1234x)^{\frac{10}{x}}$ and finding $\lim_{x \to 0} \ln y$.

Then find $\lim_{x \to 0} y = \lim_{x \to 0} e^{\ln y} = e^{\lim_{x \to 0} \ln y}$.

$$\ln (1 - 1234x)^{\frac{10}{x}} = \frac{10}{x} \ln (1 - 1234x) = \frac{10 \ln (1 - 1234x)}{x}$$ which, as $x \to 0$, is the indeterminate form $\frac{0}{0}$.

$$\lim_{x \to 0} \frac{10 \ln (1 - 1234x)}{x} \underset{\text{LH}}{=} \lim_{x \to 0} \frac{10 \cdot \frac{1}{(1 - 1234x)} \cdot \left(\frac{d}{dx}(1 - 1234x)\right)}{1} = \lim_{x \to 0} \frac{10}{(1 - 1234x)} \cdot (-1234)$$

$$= \lim_{x \to 0} \frac{-12340}{(1 - 1234x)} = \lim_{x \to 0} \frac{-12340}{(1 - 0)} = -12340.$$

We now find $\lim_{x \to 0} y = \lim_{x \to 0} e^{\ln y} = e^{\lim_{x \to 0} \ln y} = \boxed{e^{-12340}}$.