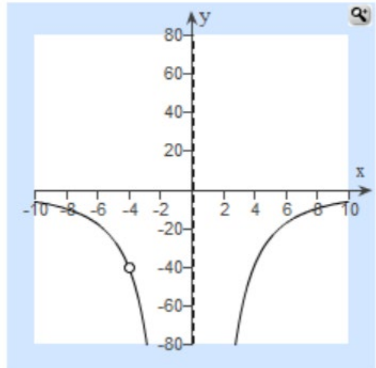
Find a formula for a rational function $r(x)$ in the form $y = \frac{p(x)}{q(x)}$ where is the $q(x)$ is the lowest polynomial degree possible, with the following properties.

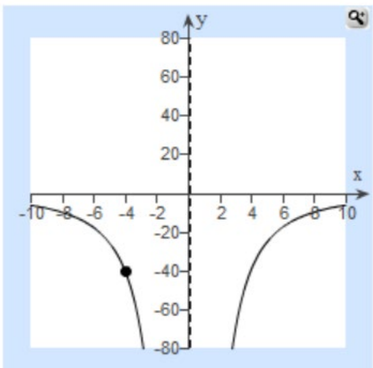


1. $\lim_{x\to 0} r(x) = -\infty$
2. $\lim_{x\to -4} r(x) = -40$
3. $r(-4)$ is undefined.
4. $\lim_{x\to \pm\infty} r(x) = 0$

A graph is shown.

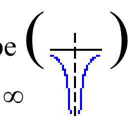
Note that the vertical asymptote is $x = 0$ and the hole is at $(-4, -40)$.

Step 1: Write a formula of the function without the hole.



Step 2: Incorporate the location of the vertical asymptote into the formula.
A vertical asymptote occurs at $x = h$ if the denominator has a factor $(x - h)$ and the numerator does not.

Step 3: Incorporate the behavior near the vertical asymptote into the formula.

Near the vertical asymptote $x = 0$, the shape  of the graph indicates the power of the factor is **even**.
Mathematically, this is written $\lim_{x\to 0} r(x) = -\infty$

which indicates that both from the left ($\lim_{x\to 0^-} r(x) = -\infty$) and from the right ($\lim_{x\to 0^+} r(x) = -\infty$)

the function gets more and more negative.
Because $q(x)$ must be the smallest degree possible, the degree of the denominator must be **2**.

So this means the formula is of the form $y = \frac{k}{x^2}$ (and in fact we expect k to be negative based on the graph.)

Step 4: Find the formula by plugging in the point $x = -4$, $y = -40$ and solve for k.

$$y = \frac{k}{x^2}$$
$$-40 = \frac{k}{(-4)^2}$$
$$-40 = \frac{k}{16}$$
$$k = -40 \cdot 16$$
$$= -640$$

Thus $y = \frac{-640}{x^2}$**.** (Check with a grapher that it passes through the point $x = -4$, $y = -40$ using a table feature.)

Step 5: Modify the function so it has a hole at $x = -4$. Using common factors, we have $r(x) = \frac{-640(x+4)}{x^2(x+4)}$.