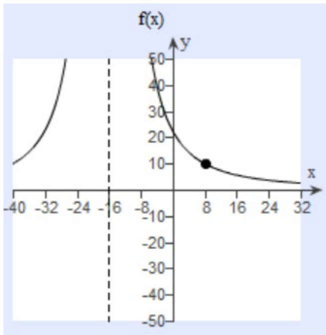
Find formulas for the rational functions below.

a. Construct a formula for the rational function $f(x)$ in the form $y = \frac{p(x)}{q(x)}$ where $q(x)$ is the lowest polynomial degree possible, with the following graph.



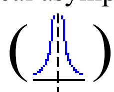
The properties of $f(x)$ are as follows:

1. $\lim_{x \to -16} f(x) = \infty$
2. $\lim_{x \to \pm\infty} f(x) = 0$
3. $f(8) = 10$

Note that the vertical asymptote is $x = -16$ and $f(x)$ passes through (8, 10).

Step 1: Incorporate the location of the vertical asymptote into the formula.
A vertical asymptote occurs at $x = h$ if the denominator has a factor $(x - h)$ and the numerator does not.

Step 2: Incorporate the behavior near the vertical asymptote into the formula.

Near the vertical asymptote $x = -16$, the shape  of the graph indicates the power of the factor is **even**. Mathematically, this is written $\lim_{x \to -16} r(x) = \infty$ which indicates that both from the left ($\lim_{x \to -16^-} r(x) = \infty$) and from the right ($\lim_{x \to -16^+} r(x) = \infty$) the function gets larger and larger. Because $q(x)$ must be the smallest degree possible, the degree of the denominator must be **2**. So this means the formula is of the form $y = \frac{k}{(x+16)^2}$

(and in fact we expect k to be positive based on the graph.)

Step 3: Find the formula by plugging in the point $x = 8$, $y = 10$ and solve for k.

$$y = \frac{k}{(x+16)^2}$$

$$10 = \frac{k}{(8+16)^2} = \frac{k}{(24)^2} = \frac{k}{576}$$

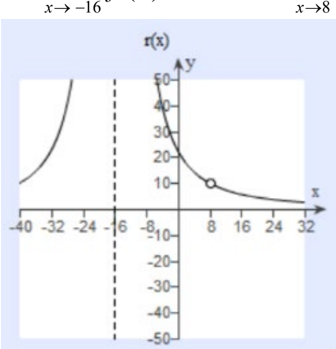
$$k = 576 \cdot 10 = 5760$$

$$y = \frac{5760}{(x+16)^2}$$

Thus $f(x) = \frac{5760}{(x+16)^2}$. (Check with a grapher that it passes through the point $x = 8$, $y = 10$ using a table feature.)

b. Construct a formula for the rational function $r(x)$ with the same properties as $f(x)$ except that $r(8)$ is undefined and $\lim_{x \to 8} r(x) = 10$. In other words

1. $\lim_{x \to -16} f(x) = \infty$ 2. $\lim_{x \to 8} r(x) = 10$ 3. $r(8)$ is undefined. 4. $\lim_{x \to \pm\infty} r(x) = 0$



Modify the function $f(x)$ so it has a hole at $x = 8$.

Using common factors, we have $r(x) = f(x) \cdot \frac{(x-8)}{(x-8)}$

$$= \frac{5760(x-8)}{(x+16)^2(x-8)}$$