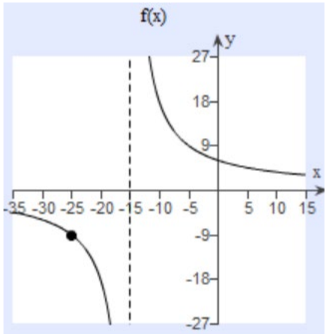
Find formulas for the rational functions below.

a. Construct a formula for the rational function $f(x)$ in the form $y = \dfrac{p(x)}{q(x)}$ where $q(x)$ is the lowest polynomial degree possible, with the following graph.



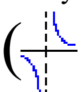
The properties of $f(x)$ are as follows:

1. $\lim_{x \to -15^-} f(x) = -\infty$ and $\lim_{x \to -15^+} r(x) = \infty$
2. $\lim_{x \to \pm\infty} f(x) = 0$
3. $f(-25) = -9$

Note that the vertical asymptote is $x = -15$ and $f(x)$ passes through $(-25, -9)$.

Step 1: Incorporate the *location* of the vertical asymptote into the formula. This corresponds to the *zero* of the factor. A vertical asymptote occurs at $x = h$ if the denominator has a factor $(x - h)$ and the numerator does not.

Step 2: Incorporate the *behavior* near the vertical asymptote into the formula. This corresponds to the *power* of the factor.

Near the vertical asymptote $x = -15$, the shape () of the graph indicates the power of the factor is **odd**.
Mathematically, this is written that from the left ($\lim_{x \to -15^-} r(x) = -\infty$) and from the right ($\lim_{x \to -15^+} r(x) = \infty$)
Because $q(x)$ must be the smallest degree possible, the degree of the denominator must be **1**.
So this means the formula is of the form $y = \dfrac{k}{x+15}$ (and in fact we expect k to be positive based on the graph.)

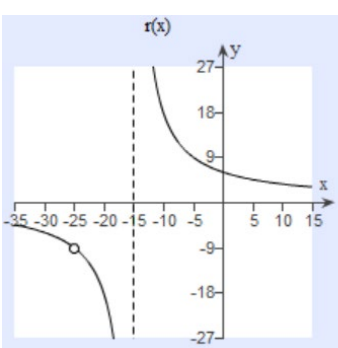
Step 3: Find the formula by plugging in the point $x = -25$, $y = -9$ and solve for k.

$$y = \frac{k}{x+15}$$
$$-9 = \frac{k}{-25+15} = \frac{k}{-10}$$
$$k = -9 \cdot 1 - 0 = 90$$
$$y = \frac{90}{x+15}$$

Thus $f(x) = \dfrac{90}{x+15}$. (Check with a grapher that it passes through the point $x = -25$, $y = -9$ using a table feature.)

b. Construct a formula for the rational function $r(x)$ with the same properties as $f(x)$ except that $r(-25)$ is undefined and $\lim_{x \to -25} r(x) = -9$. In other words:

1. $\lim_{x \to -15^-} f(x) = -\infty$ and $\lim_{x \to -15^+} r(x) = \infty$ 2. $\lim_{x \to -25} r(x) = -9$ 3. $r(-25)$ is undefined.   4. $\lim_{x \to \pm\infty} r(x) = 0$



Modify the function $f(x)$ so it has a hole at $x = -25$.

Using common factors, we have $r(x) = f(x) \cdot \dfrac{(x+25)}{(x+25)}$

$$= \frac{90(x+25)}{(x+15)(x+25)}$$