LET'S PLAY CALC-PARDY!!

http://www.magicnet.net/~itms/jeopardy/index.htm

Calc-pardy

Chain Gang	We're Related	Optimus Prime	Vital Signs	Critical Thinking
Q $200	Q $200	Q $200	Q $200	Q $200
Q $400	Q $400	Q $400	Q $400	Q $400
Q $600	Q $600	Q $600	Q $600	Q $600
Q $800	Q $800	Q $800	Q $800	Q $800
Q $1000	Q $1000	Q $1000	Q $1000	Q $1000

Final Jeopardy

Calc-pardy

Chain Gang	We're Related	Optimus Prime	Vital Signs	Critical Thinking
Q $200	Q $200	Q $200	Q $200	Q $200
Q $400	Q $400	Q $400	Q $400	Q $400
Q $600	Q $600	Q $600	Q $600	Q $600
Q $800	Q $800	Q $800	Q $800	Q $800
Q $1000	Q $1000	Q $1000	Q $1000	Q $1000

Final Jeopardy Final Jeopardy

$200 Question from Chain Gang

Find y' if $y = \ln x^9$

Simplify please.

$200 Answer from Chain Gang

$$y = \ln x^9 = 9 \ln x$$

$$y' = 9 \cdot \frac{d}{dx} \ln x$$

$$= 9 \cdot \frac{1}{x}$$

$$= \frac{9}{x}$$

OR

$$y' = \frac{d}{dx} \ln x^9 = \frac{1}{x^9} \cdot \frac{d}{dx} x^9$$

$$= \frac{1}{x^9} \cdot 9x^8$$

$$= \frac{9}{x}$$

$400 Question from Chain Gang

Find y' if

$$y = (5x^{10} + 10)^{20}$$

$400 Answer from Chain Gang

$$y = (5x^{10} + 10)^{20}$$

$$y' = 20(5x^{10} + 10)^{19} \cdot (50x^9)$$

$$= 1000x^9 (5x^{10} + 10)^{19}$$

$600 Question from Chain Gang

Find y' if $y = \tan(5x)\sec(5x^2)\cot(5x)$

$600 Answer from Chain Gang

Find y' if $y = \tan(5x)\sec(5x^2)\cot(5x)$

Simplify first

$y = \tan(5x)\sec(5x^2)\cot(5x)$ Note: $\tan(5x)\cot(5x) = 1$

$=\sec(5x^2)$

$$y' = \sec(5x^2)\tan(5x^2)\cdot\frac{d}{dx}(5x^2)$$

$$= \sec(5x^2)\tan(5x^2)\cdot(10x)$$

$800 Question from Chain Gang

Find y' if

$$y = \frac{e^{2x}}{e^x - 4}$$

$800 Answer from Chain Gang

$$y = \frac{e^{2x}}{e^x - 4}$$

$$y' = \frac{(e^x-4)\cdot\frac{d}{dx}e^{2x} - e^{2x}\cdot\frac{d}{dx}(e^x-4)}{(e^x-4)^2} = \frac{(e^x-4)\cdot 2e^{2x} - e^{2x}\cdot e^x}{(e^x-4)^2}$$

$$= \frac{2e^{2x}\cdot e^x - 8e^{2x} - e^{2x}\cdot e^x}{(e^x-4)^2}$$

$$= \frac{2e^{3x} - 8e^{2x} - e^{3x}}{(e^x-4)^2}$$

$$= \frac{e^{3x} - 8e^{2x}}{(e^x-4)^2}$$

$$= \frac{e^{2x}(e^x-8)}{(e^x-4)^2}$$

$1000 Question from Chain Gang

$$y = \tan^{-1}(e^x)$$

$$y' = ?$$

$1000 Answer from Chain Gang

$$y = \tan^{-1}(e^x)$$
$$= \tan^{-1}(u) \quad \longrightarrow \quad u = e^x$$
$$y' = \frac{1}{1+u^2} \cdot \frac{du}{dx} \qquad \frac{du}{dx} = e^x$$
$$= \frac{1}{1+(e^x)^2} \cdot e^x$$
$$= \frac{e^x}{1+e^{2x}}$$

$200 Question from We're Related

If a circle's radius increases at 6 cm / s, find the rate the area increases when the radius is 10 cm. Provide units.

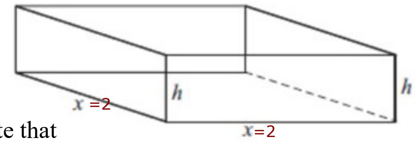
$200 Answer from We're Related

If a circle's radius increases at 6 cm / s, find the rate the area increases when the radius is 10 cm. Provide units.

$$A = \pi r^2$$
$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$
$$= 2\pi 10 \cdot 6$$
$$= 120\pi \text{ cm}^2/\text{s}$$

$400 Question from We're Related

If the height h increases at 8 cm / s, and the base x is fixed at 2 cm, find the rate that the volume increases. Provide units.



$400 Answer from We're Related

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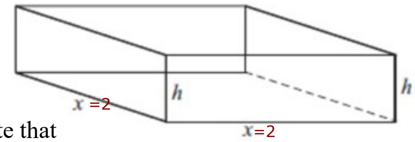
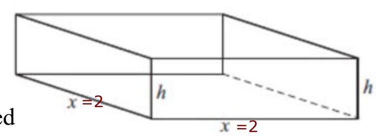


$$V = 2^2 h$$
$$V = 4h$$
$$\frac{dV}{dt} = 4 \cdot \frac{dh}{dt}$$
$$= 4 \cdot 8$$
$$= 32\,\text{cm}^3/\text{s}$$

$600 Question from We're Related

If the volume V increases at 100 cm^3 / s, and the base x is fixed at $x = 2$ cm, find the rate that the height h increases when the height $h = 10$. Provide units.



$600 Answer from We're Related

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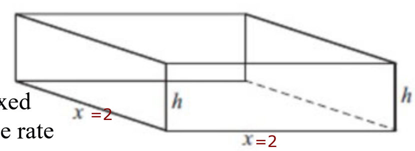
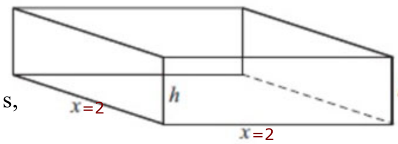


$$V = 2^2 h$$
$$V = 4h$$
$$\frac{dV}{dt} = 4\frac{dh}{dt}$$
$$100 = 4\frac{dh}{dt}$$
$$\frac{dh}{dt} = 25 \text{ cm / s}$$

Height changes at a constant rate regardless of h.

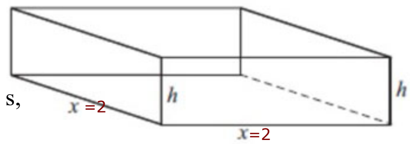
\$800 Question from We're Related

Initially the box is empty. If it is filled with water at a rate of 5 cm^3/s, and the height h is fixed at 10 cm, find the rate that the base x increases when the base $x = 2$. Provide units.



\$800 Answer from We're Related

Initially the box is empty. If it is filled with water at a rate of 5 cm^3/s, and the height h is fixed at 10 cm, find the rate that the base x increases when the base $x = 2$. Provide units.



$$V = x^2 h$$
$$= x^2 \cdot 10$$
$$= 10x^2$$

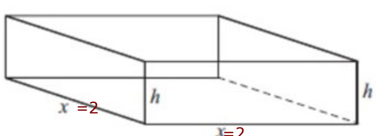
$$\frac{dV}{dt} = 20x \cdot \frac{dx}{dt}$$
$$\frac{dV}{dt} = 20x \cdot \frac{dx}{dt}$$
$$5 = 20x \cdot \frac{dx}{dt}$$
$$\frac{dx}{dt} = \frac{5}{20x} = \frac{1}{4x}$$

$$\frac{dx}{dt} = \frac{1}{4x}$$
$$= \frac{1}{4 \cdot 2} \text{ cm/s}$$
$$= \frac{1}{8} \text{ cm/s}$$

\$1000 Question from We're Related

A rectangular tank initially contains 400 cm^3 of water. If the volume decreases at 4 cm^3/s and the base x is fixed at 2 cm, write h as a function of t.

Determine the rate at which the height is changing. Provide units.



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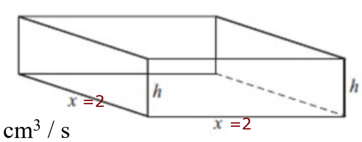


$$V = 400 - 4t$$
$$\frac{dV}{dt} = -4$$

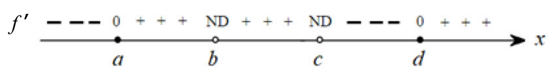
$$V = x^2 h = 2^2 h$$
$$= 4h$$
$$400 - 4t = 4h$$
$$h = \boxed{100 - t}$$
$$\frac{dh}{dt} = \boxed{-1 \text{ cm/s}}$$

OR

$V = 4h$ so at $t = 0$
$$400 = 4h$$
$$h = 100$$
$$V = 4h$$
$$\frac{dV}{dt} = 4 \cdot \frac{dh}{dt}$$
$$-4 = 4 \cdot \frac{dh}{dt}$$
$$\frac{dh}{dt} = \boxed{-1 \text{ cm/s}}$$
$$h = \boxed{100 - t}$$

$200 Question from Optimus Prime

The signs of f' are shown. For what value(s) does f have a local maximum?



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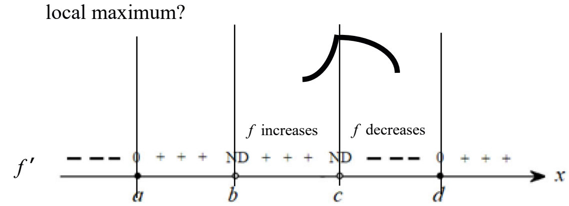
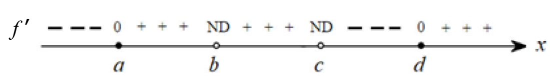


f has a local maximum at $x = c$.

$400 Question from Optimus Prime

The signs of f' are shown. For what value(s) does f has a local minimum?



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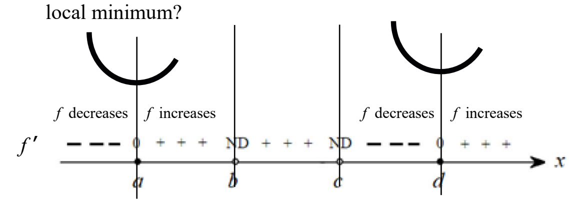
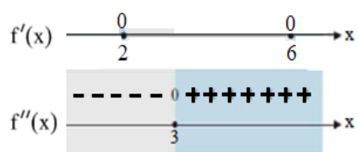


f has a local minimum at $x = a$ and $x = d$.

$600 Question from Optimus Prime

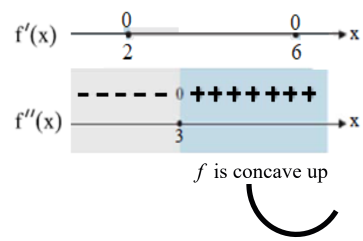
The signs of f' and f'' are shown. For what value(s) does f have a local minimum?



$600 Answer from Optimus Prime

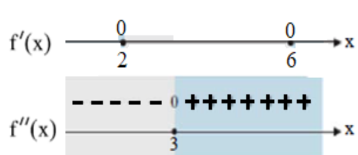
The signs of f' and f'' are shown. For what value(s) does f have a local minimum?

f has a local minimum at $x = 6$.



$800 Question from Optimus Prime

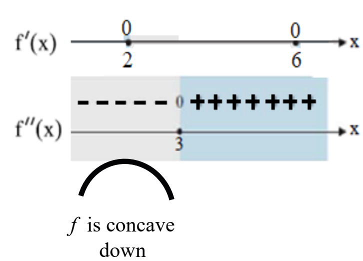
The signs of f' and f'' are shown. For what value(s) does f have a local maximum?



$800 Answer from Optimus Prime

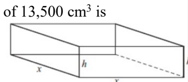
The signs of f' and f'' are shown. For what value(s) does f have a local minimum?

f has a local maximum at $x = 2$.



$1000 Question from Optimus Prime

A rectangular tank with a square base, an open top, and volume of 13,500 cm^3 is to be constructed of sheet steel. The tank with the minimum surface area has a square base with a side length of ? and height of ?



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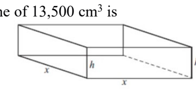


$$V = 13500$$
$$x^2 h = 13500$$
$$h = \frac{13500}{x^2}$$

$$S = 4x \cdot h + x^2$$
$$S = 4x \cdot \frac{13500}{x^2} + x^2$$
$$S = \frac{54000}{x} + x^2$$
$$S = 54000x^{-1} + x^2$$
$$S' = -54000x^{-2} + 2x$$
$$S' = -\frac{54000}{x^2} + 2x$$

$$-\frac{54000}{x^2} + 2x = 0$$
$$2x = \frac{54000}{x^2}$$
$$x^3 = \frac{54000}{2}$$
$$x^3 = 27000$$
$$x = 30$$

$$h = \frac{54000}{x^2}$$
$$h = \frac{54000}{30^2}$$
$$h = 15$$

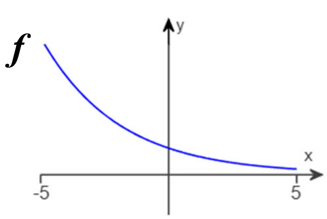
```
NORMAL FLOAT AUTO REAL RADIAN MP
Plot1  Plot2  Plot3
\Y1=54000/X+X²
\Y2=13500/X²
\Y3=-54000/X²+2X
```

base	S	height	S'
X	Y1	Y2	Y3
27	2729	18.519	⁻20.07
28	2712.6	17.219	⁻12.88
29	2703.1	16.052	⁻6.209
30	2700	15	0
31	2702.9	14.048	5.8085
32	2711.5	13.184	11.266
33	2725.4	12.397	16.413
34	2744.2	11.678	21.287
35	2767.9	11.02	25.918
36	2796	10.417	30.333
37	2828.5	9.8612	34.555

a side length of 30 and height of 15

$200 Question from Vital Signs

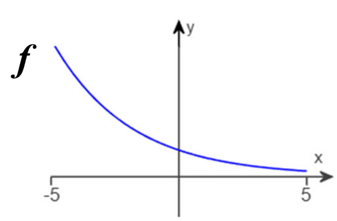
Sign of *f ″*



$200 Answer from Vital Signs

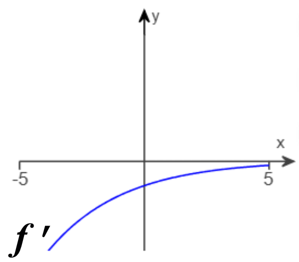
Sign of *f ″*

Since *f* is concave up, *f ″* is positive.



$400 Question from Vital Signs

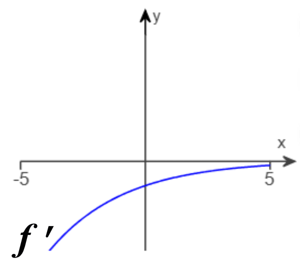
Sign of f''



$400 Answer from Vital Signs

Sign of f''

Since f' is increasing, f'' is positive



$600 Question from Vital Signs

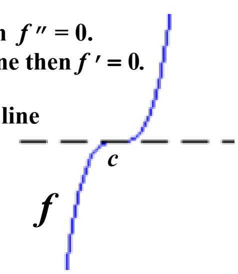
Sketch a graph of f which has a value $x = c$ where f' and f'' are both 0

$600 Answer from Vital Signs

Sketch a graph of f which has a value $x = c$ where f' and f'' are both 0

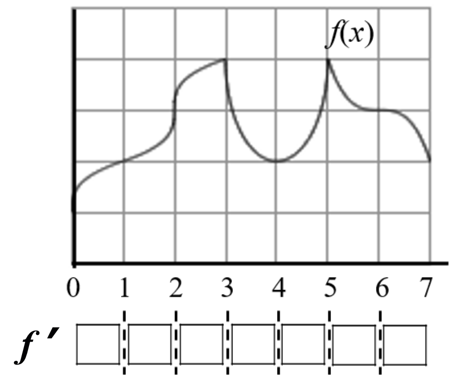
If f has an inflection point then $f'' = 0$.
If f has a horizontal tangent line then $f' = 0$.

Alternately, if f is a horizontal line then both $f' = 0$ and $f'' = 0$.



$800 Question from Vital Signs

Put + or – in each box



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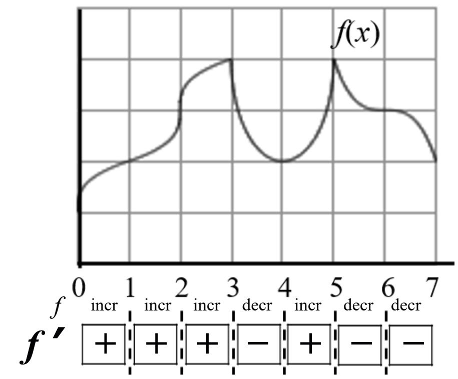

$1000 Question from Vital Signs

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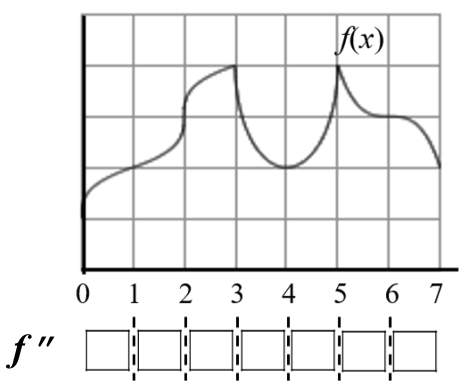

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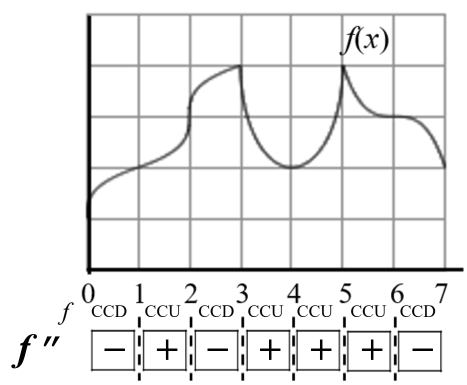
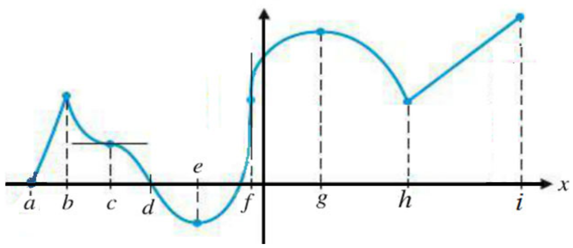


$200 Question from Critical Thinking

List all the critical values of the function



$200 Answer from Critical Thinking

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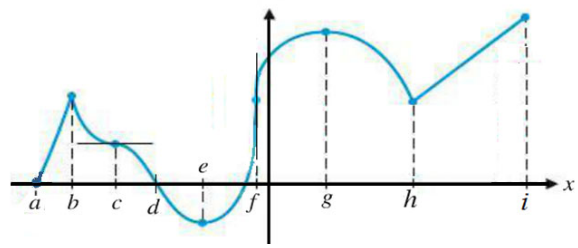
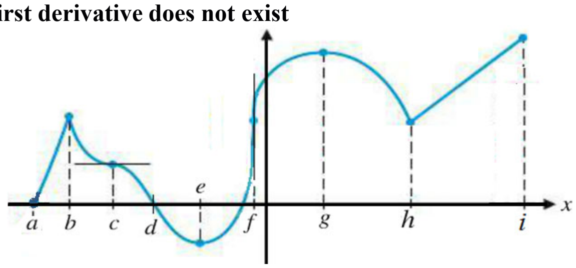


The function has critical values at $x = b, c, e, f, g, h$

$400 Question from Critical Thinking

List all the critical values of the function where the first derivative does not exist



$400 Answer from Critical Thinking

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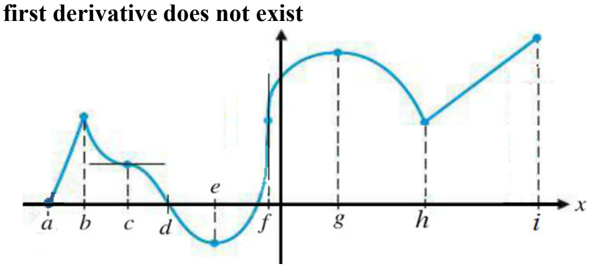
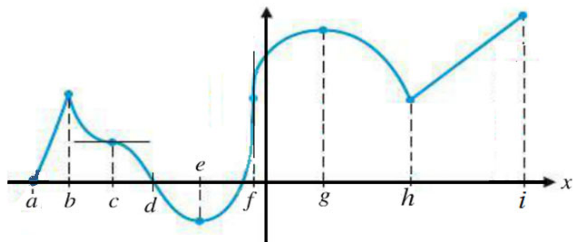


The first derivative does not exist at $x = b, f,$ and h

$600 Question from Critical Thinking

Which critical values correspond to **neither** local minima or local maxima?



$600 Answer from Critical Thinking

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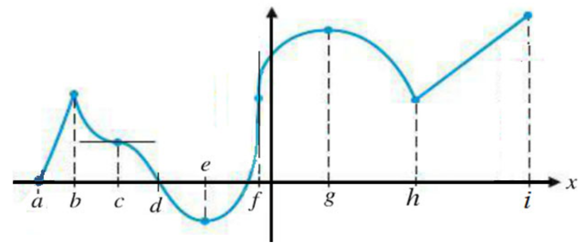


At $x = c$ and $x = f$

$800 Question from Critical Thinking

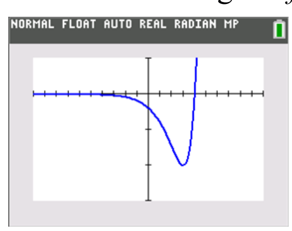
$f(x) = e^x(x-4)$ has critical value at $x = 3$.

Determine the sign of $f''(3)$.

$800 Answer from Critical Thinking

$f(x) = e^x(x-4)$ has critical value at $x = 3$.

Determine the sign of $f''(3)$.



At $x = 3$ the graph has a minimum and f is concave up so $f''(3)$ is positive.

$1000 Question from Critical Thinking

$f(x)=e^x(x-4)$ Find where $f''(x)=0$.

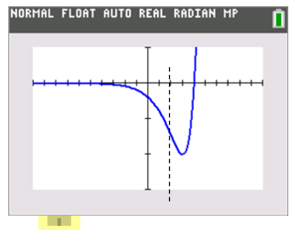
$1000 Answer from Critical Thinking

$f(x)=e^x(x-4)$ Find where $f''(x)=0$.

$f(x)=xe^x-4e^x$

$f'(x)=(x\cdot\frac{d}{dx}e^x+e^x\cdot\frac{dx}{dx})-4e^x=xe^x+e^x-4e^x=xe^x-3e^x$

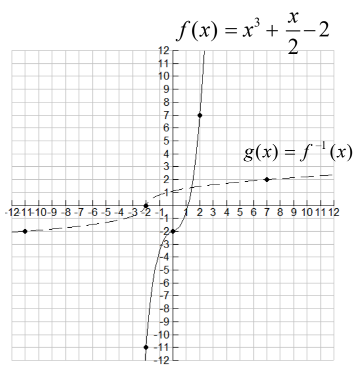
$f''(x)=\frac{d}{dx}xe^x-\frac{d}{dx}3e^x=(xe^x+e^x)-3e^x=xe^x-2e^x=e^x(x-2)$



At $x=2$ the graph changes concavity and $f''(2)=0$.

Double Jeopardy Question

Find $g'(7)$



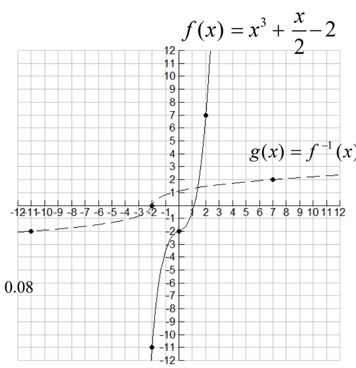
Double Jeopardy Answer

Find $g'(7)$

$f'(x)=3x^2+\frac{1}{2}$

$f'(2)=3\cdot 2^2+\frac{1}{2}=12.5$ or $\frac{25}{2}$

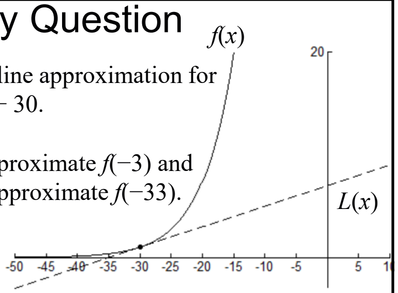
$g'(7)=\frac{1}{f'(2)}=\frac{1}{12.5}$ or $\frac{2}{25}$ or 0.08



Final Jeopardy Question

$L(x)$ is the tangent line approximation for $f(x) = e^{0.2x+6}$ at $x = -30$.

$L(-3)$ is used to approximate $f(-3)$ and $L(-33)$ is used to approximate $f(-33)$.



Which of these $L(-3)$ or $L(-33)$, is a better approximation of the function value?

Report the formula for $L(x)$ and the full values of both $L(-3)$ and $L(-33)$.

Final Jeopardy Answer

$L(x)$ is the tangent line approximation for $f(x) = e^{0.2x+6}$ at $x = -30$.

If $f(x) = e^{0.2x+6}$ then $f(-30) = e^0 = 1$

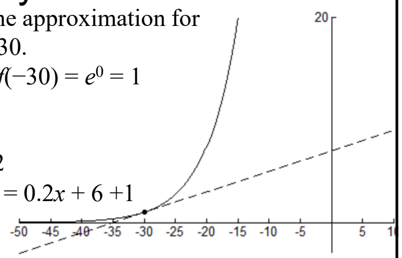
and $f'(x) = 0.2e^{0.2x+6}$

so

$f'(-30) = 0.2e^0 = 0.2$

$L(x) = 0.2(x+30) + 1 = 0.2x + 6 + 1$

$= \mathbf{0.2x + 7}$



$L(-33) = 0.4$ and $f(-33) \approx \mathbf{0.548812}$

$L(-3) = 6.4$ and $f(-3) \approx \mathbf{221.4}$

So $L(-33)$ is a better approximation of the function value.

The graph helps you see this by inspection.