

Find the derivative of  $y = \frac{(x+5)}{8x^{20}e^{11x}}$  using logarithmic differentiation.

1. Take the natural logarithm (ln) of both sides of the equation.

$$\ln y = \ln \frac{(x+5)}{8x^{20}e^{11x}}$$

2. Use logarithmic properties to simplify the expression:

$$\ln y = \ln \frac{(x+5)}{8x^{20}e^{11x}} \quad \text{Quotient to Difference Property}$$

$$\ln y = \ln(x+5) - \ln(8x^{20}e^{11x}) \quad \text{Product to Sum Property}$$

$$\ln y = \ln(x+5) - \ln 8 - \ln x^{20} - \ln e^{11x} \quad \text{Power to Product Property or Bob Barker Property}$$

$$\ln y = \ln(x+5) - \ln 8 - 20 \ln x - \ln e^{11x} \quad \text{Inverse Property}$$

$$\ln y = \ln(x+5) - \ln 8 - 20 \ln x - 11x$$

3. Differentiate both sides with respect to  $x$  using implicit differentiation. Use the chain rule.

$$\frac{d}{dx} \ln y = \frac{d}{dx} \ln(x+5) - \frac{d}{dx} \ln 8 - 20 \frac{d}{dx} \ln x - 11 \frac{d}{dx} x$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x+5} - 0 - \frac{20}{x} - 11$$

4. Solve for  $\frac{dy}{dx}$  and substitute the expression for  $y$ :

$$\frac{dy}{dx} = y \left( \frac{1}{x+5} - \frac{20}{x} - 11 \right)$$

$$\frac{dy}{dx} = \frac{(x+5)}{8x^{20}e^{11x}} \left( \frac{1}{x+5} - \frac{20}{x} - 11 \right)$$

5. Simplify the expression:

$$\frac{dy}{dx} = \frac{(x+5)}{8x^{20}e^{11x}} \left( \frac{1}{x+5} \right) - \frac{(x+5)}{8x^{20}e^{11x}} \left( \frac{20}{x} \right) - \frac{(x+5)}{8x^{20}e^{11x}} \cdot 11 \quad \text{Distribute}$$

$$\frac{dy}{dx} = \frac{1}{8x^{20}e^{11x}} - \frac{20(x+5)}{8x^{21}e^{11x}} - \frac{11(x+5)}{8x^{20}e^{11x}} \quad \text{Simplify } \frac{x+5}{x+5} \text{ with "1 out"}$$

$$\frac{dy}{dx} = \frac{x}{x} \cdot \frac{1}{8x^{20}e^{11x}} - \frac{20(x+5)}{8x^{21}e^{11x}} - \frac{x}{x} \cdot \frac{11(x+5)}{8x^{20}e^{11x}} \quad \text{Get common denominators with "1 in"}$$

$$\frac{dy}{dx} = \frac{x-20x-100-11x^2-55x}{8x^{21}e^{11x}} \quad \text{Combine numerators and distribute.}$$

$$\frac{dy}{dx} = \frac{-11x^2-74x-100}{8x^{21}e^{11x}} \quad \text{Combine like terms.}$$