

U Got This

Complete the table. Make the nasty integrand pretty.

Nasty Integrand	u	du	$\int f(u)du$	Result in terms of x
$\int x(x-5)^{10} dx$	$u = x-5$ \downarrow $x = u+5$	$du = dx$	$\int x(x-5)^{10} dx$ $= \int (u+5) u^{10} du$ $= \int (u^{11} + 5u^{10}) du$	$\int (u^{11} + 5u^{10}) du$ $= \int u^{11} du + 5 \int u^{10} du$ $= \frac{1}{12} u^{12} + \frac{5}{11} u^{11} + C$ $= \frac{1}{12} (x-5)^{12} + \frac{5}{11} (x-5)^{11} + C$
$\int \frac{x}{x-3} dx$	$u = x-3$ \downarrow $x = u+3$	$du = dx$	$\int \frac{x}{x-3} dx$ $= \int \frac{u+3}{u} du$ $= \int (\frac{u}{u} + \frac{3}{u}) du$ $= \int (1 + \frac{3}{u}) du$	$\int (1 + \frac{3}{u}) du = \int du + 3 \int \frac{du}{u}$ $= u + 3 \ln u + C$ $= x-3 + 3 \ln x-3 + C$
$\int \frac{x}{\sqrt{x-100}} dx$	One way: $u^2 = x-100$ \downarrow $x = u^2 + 100$ $u = \sqrt{x-100}$ or $(x-100)^{1/2}$	$u^2 = x-100$ $2u du = dx$	$\int \frac{x}{\sqrt{x-100}} dx$ $= \int \frac{u^2+100}{u} \cdot 2u du$ $= \int 2(u^2+100) du$	$2 \int u^2 du + 200 \int du$ $= 2 \cdot \frac{1}{3} u^3 + 200u + C$ $= \frac{2}{3} (x-100)^{3/2} + 200\sqrt{x-100} + C$
$\int \frac{x}{\sqrt[3]{x-2}} dx$	One way: $u^3 = x-2$ \downarrow $x = u^3 + 2$ $u = (x-2)^{1/3}$	$u^3 = x-2$ $3u^2 du = dx$	$\int \frac{x}{\sqrt[3]{x-2}} dx$ $= \int \frac{u^3+2}{u} \cdot 3u^2 du$ $= \int 3(u^3+2)u du$	$3 \int u^4 du + \int 6u du$ $= \frac{3}{5} u^5 + \frac{6}{2} u^2 + C$ $= \frac{3}{5} (x-2)^{5/3} + 3(x-2)^{2/3} + C$
$\int \sqrt[5]{(3x+2)(3x+2)} dx$ $= \int (3x+2)^{6/5} dx$ $= \int (3x+2)^{6/5} dx$	One way: $u = 3x+2$ $du = 3dx$ Another way: $u^5 = 3x+2$ $5u^4 du = 3dx$ \downarrow $dx = \frac{5}{3} u^4 du$ $u = (3x+2)^{1/5}$	$u = 3x+2$ $du = 3dx$ $u^5 = 3x+2$ $5u^4 du = 3dx$ \downarrow $dx = \frac{5}{3} u^4 du$ $u = (3x+2)^{1/5}$	$\frac{1}{3} \int (3x+2)^{6/5} 3 dx$ $= \frac{1}{3} \int u^{6/5} du$ $\int \sqrt[5]{3x+2} \cdot (3x+2) dx$ $= \int u \cdot u^5 \cdot \frac{5}{3} u^4 du$ $= \int \frac{5}{3} u^{10} du$	$\frac{1}{3} \int u^{6/5} du$ $= \frac{1}{3} \cdot \frac{5}{11} u^{11/5} + C$ $= \frac{5}{33} (3x+2)^{11/5} + C$ $\frac{5}{3} \cdot \frac{1}{11} u^{11} + C = \frac{5}{33} (3x+2)^{11/5} + C$
$\int 49x(7x+25)^8 dx$	$u = 7x+25$ \downarrow $x = \frac{u-25}{7}$	$du = 7dx$ \downarrow $dx = \frac{du}{7}$	$\int 49x(7x+25)^8 dx$ $= \int 49 \cdot \frac{(u-25)}{7} \cdot u^8 \cdot \frac{du}{7}$ $= \int \frac{49}{7 \cdot 7} \cdot (u-25) u^8 du$ $= \int (u^9 - 25u^8) du$	$\int (u^9 - 25u^8) du$ $= \frac{1}{10} u^{10} - \frac{25}{9} u^9 + C$ $= \frac{1}{10} (7x+25)^{10} - \frac{25}{9} (7x+25)^9 + C$