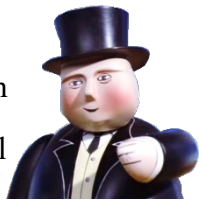
Thomas the Tank Engine's Velocity Gets Mean

Thomas the Tank Engine is $d = f(t) = \frac{t^3}{3} - 3t^2 + \frac{14t}{3} + 10$ miles from his boss Sir Topham Hatt, where t is given in hours. The graph of $d = f(t)$ is given below for $0 \le t \le 7$ as well as a line segment to represent the average rate of change $\frac{\Delta d}{\Delta t}$ of $f(t)$ on the graph of $d = f(t)$ for each interval. Note: The word average used in this sense is also called the **mean** velocity.



1. From $t = 0$ to $t = 7$: $\Delta d = 0$ miles, $\Delta t = 7$ hr, and

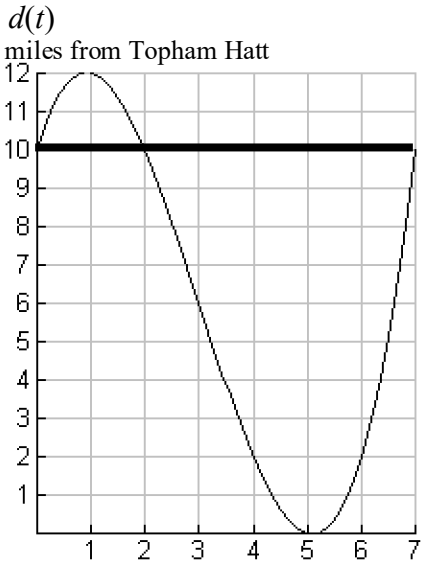
   his average rate of change from $t = 0$ to $t = 7$ is $\frac{\Delta d}{\Delta t} =$ **0** mph.

   Is Thomas' instantaneous rate of change ever equal to his average (or mean) rate of change from $t = 0$ to $t = 7$? YES / NO

   If yes how many times? _____

   Approximately what value(s) of t? _____(Round to an integer.)

   Interpret what this means to Thomas.



Make an appropriate sketch on the graph to show what you have claimed.
Fun fact: **Bhaskara II**, India, 12th Century and **Michel Rolle**, France, 1691 made the same claim.

2. From $t = 2$ to $t = 5$: $\Delta d = -10$ miles, $\Delta t = 3$ hr, and

   his average rate of change from $t = 2$ to $t = 5$ is $\frac{\Delta d}{\Delta t} = \frac{-10}{3}$ mph.

   Is Thomas' instantaneous rate of change ever equal to his average (or mean) rate of change from $t = 2$ to $t = 5$? YES / NO

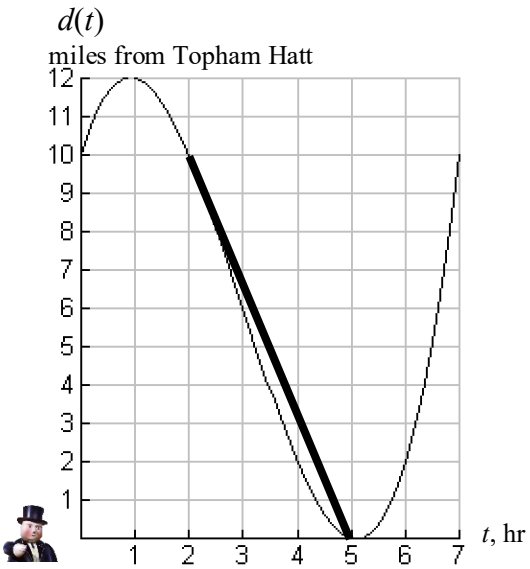
   If yes how many times? _____

   At what value(s) of t? _____

   $v = d'$ = _____

   TIP: Enter d and v in a grapher and solve graphically.

   Optional: Enter the secant line $y = \frac{-10}{3}(x - 5)$.



Use Draw Tangent
to sketch on your grapher any tangent line(s) to d
which have this slope (or enter any equations in Y=) to see any parallel lines.

Then sketch any parallel lines tangent to d on the graph above to show what you have claimed.
Ghostbuster **Gus Cauchy**, France, 1823, made this claim. It is not called Cauchy's Theorem because of these.