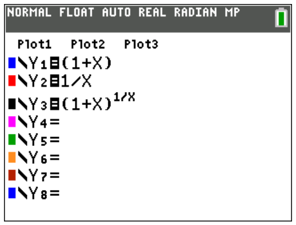
The Derivative of the Natural Logarithm Function $y = \ln(x)$

Complete the steps to show why $\frac{d}{dx}\ln x = \frac{1}{x}$ to earn +1 Rhino Participation Bonus!

1. Fun Fact: The value of $\lim_{Q\to 0}(1+Q)^{\frac{1}{Q}}$ is a famous number. What is the **exact** value of this limit? Fill it in the box below.

   You can explore this limit with a graphing calculator as shown below.



NORMAL FLOAT AUTO REAL RADIAN MP

X	Y1	Y2	Y3	*Exact Value*
0.1	1.1	10	2.5937	$(1.1)^{10}$
0.01	1.01	100	2.7048	$(1.01)^{100}$
0.001	1.001	1000	2.7169	$(1.001)^{1000}$
0.001	1.0001	10000	2.7181	$(1.0001)^{10000}$
0.0001	1.00001	100000	2.7183	$(1.00001)^{100000}$

   Complete: $(1.00000000000000000000000000000000000000000000000000000000...0001)^{100000000000000000000000000000000000000000000000000000000000...000} = \lim_{Q\to 0}(1+Q)^{\frac{1}{Q}} = \boxed{}$

2. Recall the following properties of logarithms.

   a. Sum Property: $\ln A + \ln B = \boxed{}$

   b. Difference Property: $\ln A - \ln B = \boxed{}$

   c. Power Property: $k \cdot \ln A = \boxed{}$

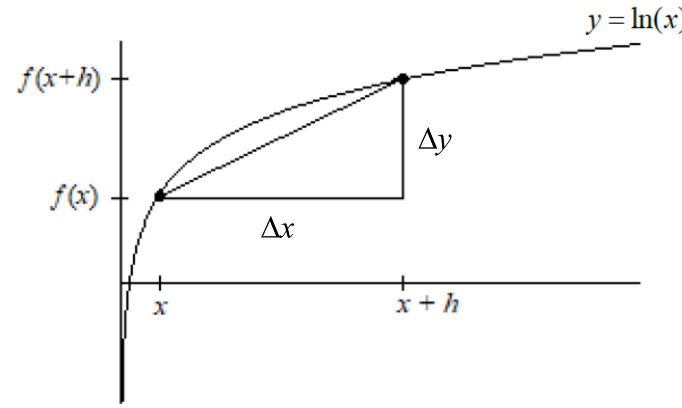
   d. Can the expression ln (A + B) be simplified? Circle one: YES NO

   If yes, please simplify it below. If not, please leave as is.

(OVER PLEASE)

3. Write in terms of the natural logarithm function and x and h.

$$\frac{d}{dx}\ln x = \lim_{h\to 0}\frac{f(x+h)-f(x)}{x+h-x} = \lim_{h\to 0}\boxed{}$$



4. Rewrite your expression in the box in #3 using the **Difference Property**.

$$= \lim_{h\to 0}\frac{\boxed{}}{h}$$

5. Use the property that $\frac{*}{h} = \frac{1}{h}\bullet *$. Follow the remaining steps. Arrows indicate you recopy the previous box.

$$= \lim_{h\to 0}\frac{1}{h}\cdot\ln\left(1+\boxed{}\right)$$ Divide your expression in the box in #4 by x.

$$= \lim_{h\to 0}\frac{1}{x}\cdot\boxed{}\cdot\ln\left(1+\boxed{}\right)$$ Rewrite $\frac{1}{h}$ using "1 in"

$$= \lim_{h\to 0}\frac{1}{x}\bullet\lim_{h\to 0}\boxed{}\cdot\ln\left(1+\boxed{}\right)$$ Write as a product of limits.

$$= \lim_{h\to 0}\frac{1}{x}\bullet\lim_{h\to 0}\boxed{}$$ Use the **Power Property**.

6. Let $Q=\frac{h}{x}$. As $h\to 0$, what happens to Q? $Q\to\boxed{}$

7. Write the limit in the last line of #5 all in terms of Q.

$$\frac{d}{dx}\ln x = \lim_{h\to 0}\frac{1}{x}\bullet\lim_{Q\to\boxed{}}\ln\left(\boxed{}\right)^{\boxed{}}$$

$$= \boxed{}\bullet\boxed{} = \boxed{}$$ Why? ______________________________