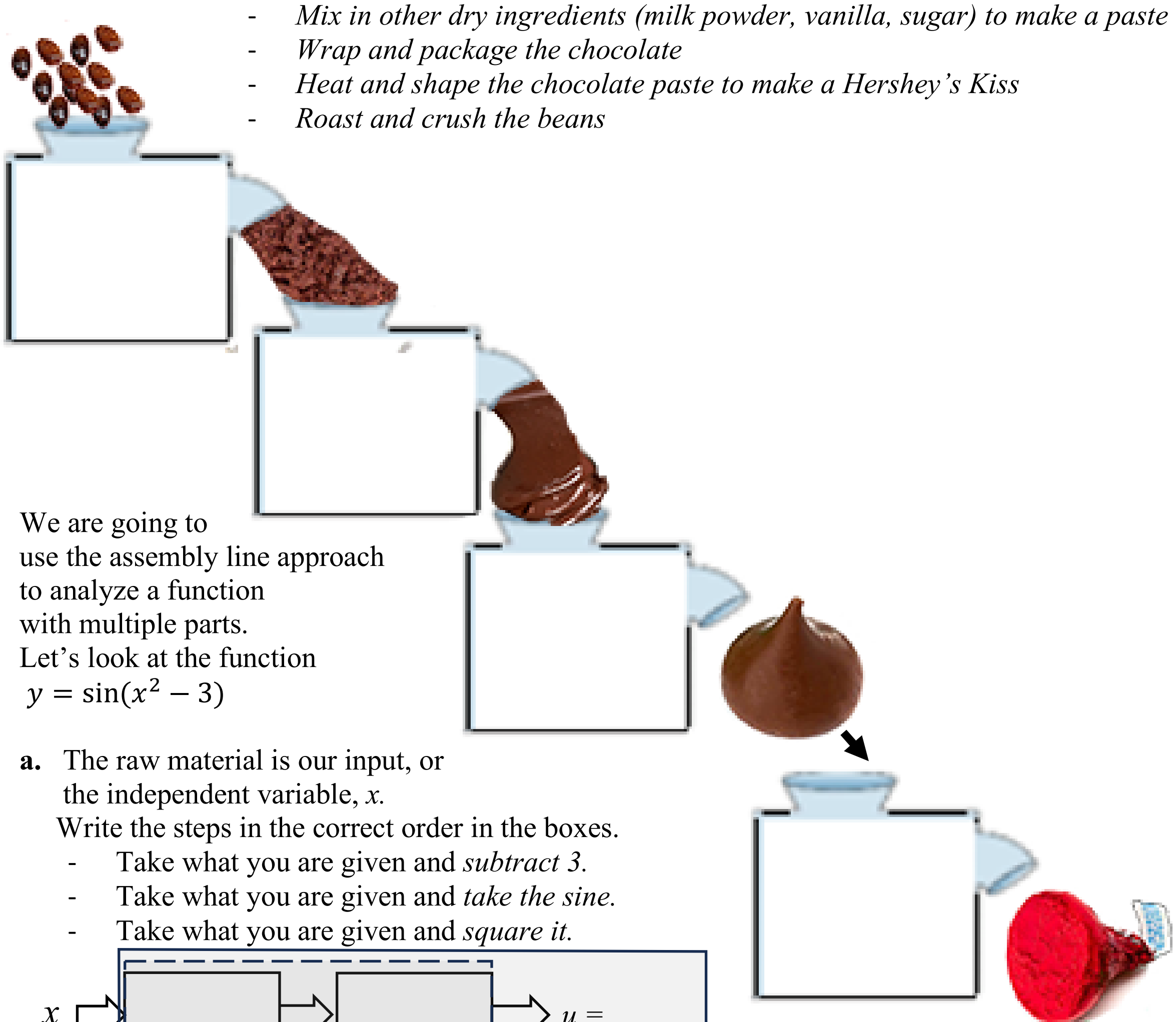
The Differentiation of a Function of a Function

The process of making chocolate is quite involved and includes [multiple steps](). In large chocolate factories, this process has many automated components and uses an assembly line. At each station, a single component of the process is completed before it is sent off to the next station in the chain. This is like function composition. You can see a function *f* composed with a function *g* is [here]() using the model of an assembly line and function machines.

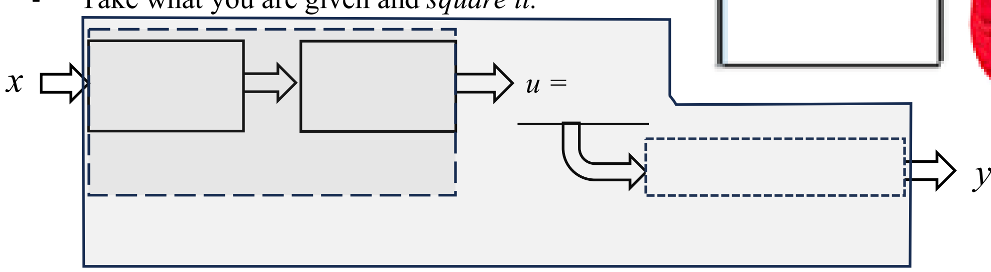
1. The steps for making chocolate kisses are jumbled below. In the ideal case, no children were harmed and used as slaves. (See https://www.slavefreechocolate.org/) Put them in the correct order by writing them on the correct function machine below. This process is prior to the acquisition of the beans.

- *Mix in other dry ingredients (milk powder, vanilla, sugar) to make a paste*
- *Wrap and package the chocolate*
- *Heat and shape the chocolate paste to make a Hershey's Kiss*
- *Roast and crush the beans*



2. We are going to use the assembly line approach to analyze a function with multiple parts. Let's look at the function $y = \sin(x^2 - 3)$

a. The raw material is our input, or the independent variable, *x*.
Write the steps in the correct order in the boxes.
- Take what you are given and *subtract 3.*
- Take what you are given and *take the sine.*
- Take what you are given and *square it.*



b. Let $u = g(x)$ be the *inside* function. Report *u* as a function *g* of *x*. $u = g(x) =$ __________. Write this in the blank in the machine.

c. Let $y = f(u)$ be the *outside* function. Report *y* as a function *f* of *u*. $y = f(u) =$ __________.

3. To find the derivative of this function, we will need to consider how each link in the chain contributes to the end product.

a. $\frac{du}{dx} = \frac{d}{dx} g(x) = \square$ Involves *x*.

b. $\frac{dy}{du} = \frac{d}{du} f(u) = \square$ Involves *u*.

c. $\frac{dy}{dx} = \frac{d\square}{d\square} \cdot \frac{d\square}{d\square}$ Insert *y*, *u*, and *x* in the boxes. Keep in mind parts **a** and **b**.

d. Use parts **a** and **b** to find the end product $\frac{dy}{dx}$ in terms of *x*.

$\frac{dy}{dx} =$ ______________________. ← Involves *x*.

Section 3.7 The Chain Rule

Important Ideas:

Check Your Understanding!

1. Let $f(x) = e^{3x^2-7x}$. Find $f'(x)$.

2. If $g(x) = \cos^3(4x)$, find $\frac{dg}{dx}$.

3. Multiple Choice! Find the derivative of $y = \frac{-7}{(2x-3)^3}$

   $A) y' = \frac{42}{(2x-3)^2}$

   $B) y' = 42(2x-3)^2$

   $C) y' = \frac{42}{(2x-3)^4}$

   $D) y' = 21(2x-3)^2$

   $E) y' = \frac{21}{(2x-3)^4}$

4. Selected values of two differentiable functions, *f* and *g,* are given in the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

   a. Let $k(x) = f(g(x))$. Find $k'(3)$.

   b. Write an equation for the line tangent to k at $x = 3$.