Section 3.10 — Derivatives of Inverse Functions

Important Ideas:

If f and g are inverses of each other

$f(a)=b \Leftrightarrow g(b)=a$

We call (a,b) and (b,a) corresponding points
on f on g

Slopes: $g'(b)=\frac{1}{f'(a)}$

We can write this using inverse notation

Let $f^{-1}=g$
Then $f^{-1}(b)=a$ since $g(b)=a$
$g'(b)=\frac{1}{f'(a)}$
$(f^{-1})'(b)=\frac{1}{f'(f^{-1}(b))}$

Meh!

Check Your Understanding!

1. For $f(x)=3x+6$, find $(f^{-1})'(x)$. You do not need to find $f^{-1}(x)$.

$(f^{-1})'(x)=\boxed{\frac{1}{3}}$ For any value a, $f'(a)=3$ since f is linear. Since f is linear, f^{-1} is linear

2. Let $f(x)=x^3+x$. If $g(x)=f^{-1}(x)$ and $g(2)=1$, what is the value of $g'(2)$?

$(2,1)$ on $g \Leftrightarrow (1,2)$ on f They are corresponding points.
$g'(2)=\frac{1}{f'(1)}$ Since $f(x)=x^3+x$
then $f'(x)=3x^2+1$ and $f'(1)=4$ so $g'(2)=\boxed{\frac{1}{4}}$

3. The table below gives selected values for a differentiable and decreasing function f and its derivative. If $f^{-1}(x)$ is the inverse function of f, what is the value of $(f^{-1})'(2)$?

x	$f(x)$	$f'(x)$
0	49	0
1	2	–8
2	–1	–80

Let $f^{-1}=g$. We want $g'(2)$.
We need the corresponding point
$g(2)=b \Leftrightarrow f(b)=2$
so $b=1$ by the table.
$(1,2)$ on $f \Leftrightarrow (2,1)$ on g
$g'(2)=\frac{1}{f'(1)}=\frac{1}{-8}=\boxed{-\frac{1}{8}}$

4. Suppose that g is the inverse function of $f(x)=3x^5+6x^3+4$. Find $g'(13)$. TIP: Use a table or a graph.

using table from grapher

x	$f(x)$
0	4
1	13

$(1,13)$ on $f \Leftrightarrow (13,1)$ on g so $g'(13)=\frac{1}{f'(1)}$

$f(x)=3x^5+6x^3+4$
$f'(x)=15x+18x^2$
$f'(1)=15+18=33$

$g'(13)=\boxed{\frac{1}{33}}$

5. Find the equation of the tangent line to the *inverse* of $f(x)=x^5+2x^3+x-4$ at the point $(-4,0)$.

By substitution, $f(0)=-4$
so if g is f^{-1}, then $g(-4)=0$
We want $g'(-4)=\frac{1}{f'(0)} \Rightarrow f'(x)=5x^4+8x^2+1$
$f'(0)=1$
$g'(-4)=\frac{1}{1}=1$

The equation of the tangent line to $g(x)$ at $(-4,0)$ has slope 1
$y=1(x+4)$
or $\boxed{y=x+4}$